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Abstract

We investigate the interaction between environmental quality and fertility in an altruistic bequest model with pollution externalities created by the aggregate production. Despite the negative externality related to the endogenous childbearing decisions, the parents may choose to have fewer children in the competitive economy than in the social optimum. To achieve optimality, positive taxes on childbearing are required even with an insufficient number of children, if the social discount factor equals the parents' degree of altruism. On the other hand, child allowances constitute the optimal policy if the social discount factor exceeds the parents' degree of altruism and is assigned equally between parents and children.

Keywords Externality; Environmental quality; Fertility; Altruism; Bequest; Child allowances

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1. Introduction

In this paper, we study the relation between fertility and environmental quality. While the world population has increased rapidly, many developed countries such as Japan, Germany and Italy are faced with infertility and (future) population decline. On the other hand, even in developed countries, although the environmental concern is relatively high, polluting emissions are not necessarily controlled perfectly. Specifically, Japan has been struggling for but has not succeeded in reducing greenhouse gas emissions, while stabilization of the emissions has been achieved in several European countries. In addition, most developed countries are still suffering from various types of local environmental problems such as water pollution in rivers, lakes and sea areas, along with soil contamination.

The effect of environmental deterioration on fertility may depend on the stage of economic development. As argued in Nerlove (1991), in the early stage of development, fertility is likely to react positively to environmental deterioration, because parents are induced to have a greater number of children as environmental quality lowers under the following assumptions: (1) infant mortality rate is high and increases as environmental quality lowers; (2) "work or income utility" and "old age security utility" are important factors in fertility decisions.[2] As economic development proceeds and per capita income increases, both infant mortality rate and the influence of environment quality on it decrease. At the same time, the use of child labor decreases and

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[2] Leibenstein (1973) assumes three types of utility ascribed to a child: "work or income utility," "old age security utility" and "consumption utility."

old age security systems outside the family develop, implying that work and old age security utility declines. Hence, in a later stage of economic development, parental altruism could be a more significant factor for fertility decisions. If parents are altruistic toward their children and are concerned about their children's welfare (Barro, 1974), they may choose to have fewer children in response to environmental degradation, which would leave their children worse off. In this paper we focus on the role which altruism plays in the interaction between fertility and environmental qualities, and show that the equilibrium may be characterized by insufficient fertility and excessive pollution, which several developed countries are experiencing.

Jouvet, Michel and Vidal (2000) and Jouvet, Michel and Pestieau (2000) investigate environmental issues introducing altruistic bequests. (The latter authors consider the case where altruists and non-altruists coexist.) Assuming that individuals voluntarily contribute to pollution abatement, these studies show that a market economy results in under-contribution to pollution abatement and thus an under-provision of environmental quality due to the free rider problem. In these models, bequests also create environmental externalities via the production process, which lead to an over-accumulation of capital. To attain the social optimum, therefore, the government requires subsidies on contributions to pollution abatement and taxes on capital. These studies, however, assume exogenous fertility and the relationship between fertility and environmental qualities is outside their scope.

There are a few studies addressing the issues of fertility choices and environmental externalities in the presence of altruism, which include Harford

(1997, 1998) and Jöst and Quaas (2009). Harford (1998) considers a consumable capital good and a non-capital good, and consumption of the latter is assumed to create a pollution externality. While an increase in the number of children implies an increase in aggregate consumption of the polluting good, parents do not recognize such an impact of an extra child on pollution, and hence childbearing has an external effect. Harford shows that Pareto efficiency requires taxes on childbearing as well as Pigovian pollution taxes. Taxes on capital are not called for in his model, since bequests of capital do not entail externalities. Jöst and Quaas (2009) extend Harford (1997, 1998) to an optimal control model with a production system which emits pollutants. They consider two types of households: dynastic households (Barro and Becker, 1989) and "micro households" in which children leave their parents' household to form a new decision-making unit immediately after birth. In their model, two kinds of external effects, which arise from the individual decisions on polluting emissions and fertility, cause excessive total emissions and excessive population relative to socially optimal. While the pollution externality may be internalized by a Pigovian tax on emissions, the optimal population policy is different according to the type of household. Taxes on the household size are required in the case of dynastic households, while taxes on the number of children are required in the case of micro households.

In this paper we investigate the relationship between fertility and environmental qualities by assuming that production causes pollution and bequests embodied in productive capital create pollution externalities, as in

Jouvet, Michel and Vidal (2000) and Jouvet, Michel and Pestieau (2000). Since aggregate production is increasing in the population, pollution externalities of childbearing also prevail in our model. The co-existence of these two externalities leads to a result different from that obtained in the previous studies, namely, that the fertility rate determined in a market economy may be lower than the social optimum, although childbearing has a negative external effect on the environment. Parents choose the number of children so that the marginal benefit equals the marginal cost of having a child, and bequests toward each child constitute the marginal cost of a child. Thus, an increase in bequests raises the marginal cost of a child, and has a negative effect on the number of children. Therefore, if the level of bequests in the competitive equilibrium is higher than the social optimal level, this over-provision of bequests raises the private marginal cost of a child possibly to a level above its social marginal cost. In such a case, the number of children in the competitive equilibrium rather falls below the social optimum. According to our numerical examples, many combinations of plausible parameter values support an equilibrium in which the number of children is insufficient. On the other hand, we show that the level of pollution is unambiguously higher than the social optimum, whether the fertility rate is too high or too low.

We also examine what kind of policy is required to achieve social optimality. It is shown that, if the social discount factor for a child equals the private degree of altruism, the government needs to tax both childbearing and inheritance so as to restore efficiency, even if the fertility rate is lower than its social optimal level. This is because an over-accumulation of capital is a necessary condition for under-production of children. Once capital is adjusted to its optimal level by inheritance taxes, the factor in the under-production of children disappears, and the fertility rate exceeds its social optimal level due to its environmental externalities. On the other hand, if the social discount factor for a child is higher than the private degree of altruism, child allowances and/or subsidies on inheritance may be required to attain the social optimum. However, the optimal policy never involves a combination of taxes on childbearing and subsidies on inheritance. Furthermore, our numerical examples suggest that a combination of child allowances and inheritance taxes, which is adopted in many developed countries, is consistent with social optimality, as long as the weight to private utility in the social welfare function does not differ greatly among generations.

The rest of the paper is organized as follows. Section 2 presents a model, and characterizes the competitive equilibrium. Section 3 characterizes the social optimum, and compares it to competitive equilibrium in the benchmark case where the social discount factor equals to the private degree of altruism. Section 4 examines what kind of policy is required to decentralize the social optimum. Section 5, assuming that the social discount factor differs from private degree of altruism, reexamines the result obtained in the previous sections. Section 6 provides the conclusions.

2. The Basic Model

Suppose that there are two periods and two generations. The parents' generation (generation 0) lives for period 0 and the children's generation (generation 1) lives for period 1, with no overlapping of the periods. Each member of the same generation is identical. The population of generation 0 is N, and each member of generation 0 produces n children.

As in Becker and Barro (1988), the parents decide to have n children because they are altruistic toward their children in that each child's welfare directly enters their utility functions. It is assumed that each child costs $\beta\ (>0)$, so that $n\beta$ is the total cost of raising children. The parents allocate the remaining income after they have paid the cost of raising children between their own consumption and bequests toward their children. We also assume that the inheritance from the former generation determines the income of each generation.

The parents derive disutility from the level of pollution while deriving utility from consumption and their children's welfare. Their utility function is thus defined by

(1) $$U_0(c_0,\ \pi_0,\ n,\ U_1) = u_0[(1+r)b_0 - n(b_1+\beta)] - V_0(\pi_0) + n\delta(n)U_1,$$

where $c_0 (= (1+r)b_0 - n(b_1+\beta))$ is their consumption, b_0 is the inheritance they receive, b_1 is the bequests to each child, r is the interest rate, π_0 is the level of pollution in period 0, U_1 is the utility of each child, and $\delta(n)$ is the weight attached to each child's utility. We assume that $u_0' > 0$, $u_0'' < 0$, $V_0' > 0$, $V_0'' > 0$, $0 < \delta(n) < 1$, $\delta'(n) < 0$, $\delta(n) + \delta'(n)n > 0$ and $2\delta'(n) + \delta''(n)n < 0$.

The children consume the inheritance from their parent, and their utility function is defined by

(2) $$U_1(c_1,\ \pi_1) = u_1[(1+r)b_1] - V_1(\pi_1),$$

where π_1 is the level of pollution in period 1. We assume that $u_1' > 0$, $u_1'' < 0$, $V_1' > 0$ and $V_1'' > 0$.

We assume that the level of pollution in each period is a linear function of current production Y_i, and that no pollutants survive the period. We thus have

(3) $$\pi_i = \alpha Y_i;\ \ \alpha > 0,\ \ i = 0,\ 1.$$

Assuming a linear technology, we define the production function as

(4) $$Y_i = AK_i;\ \ A > 0,\ \ i = 0,\ 1,$$

where K_i is the stock of capital in period i.

Equilibrium on the capital market implies

(5) $$b_i = k_i;\ \ i = 0,\ 1,$$

where $k_0 \equiv K_0 / N$ and $k_1 \equiv K_1 / nN$. At equilibrium the rate of interest is equal to the marginal productivity of the capital net of depreciation:[3]

(6) $$1 + r = A.$$

We hereafter denote k_1 as k for notational simplicity.

The parents are assumed not to recognize that producing children and bequeathing their wealth to their children should degrade the future environment *via* the production process. Given π_1 as well as $(1+r)b_0$, β and π_0, therefore, the parents choose the number of children and the level of bequests so as to maximize (1). Substituting (3), (4), (5) and (6) into the

[3] We assume total depreciation after one period.

first-order conditions yields

(7) $$F(k,\ n) \equiv -nu_0'[Ak_0 - n(k+\beta)] + n\delta(n)Au_1'(Ak) = 0,$$

(8) $$G(k,\ n) \equiv -(k+\beta)u_0'[Ak_0 - n(k+\beta)] + [\delta(n) + n\delta'(n)]\{u_1(Ak) - V_1(\alpha ANnk)\} = 0\,.$$

The competitive equilibrium (k^*, n^*) is characterized by (7) and (8).

We define $G(k,\ n)$ in (8) as the private marginal net benefit (PMNB) of a child (Similarly, $F(k,\ n)$ in (7) as PMNB of bequests). The first term in (8) is the marginal disutility from the decrease in parental consumption by having an additional child, and represents the private marginal cost of a child. The second term in the RHS of (8) is the increase in parental utility derived from altruism when adding an additional child, and represents the private marginal benefit of a child.

3. Social Optimum

In this section, we characterize the social optimum, and compare it to the competitive equilibrium obtained in the previous section. In particular, we show that the number of children chosen may be lower in the competitive equilibrium than in the social optimal allocation, albeit children create negative environmental externalities.

3.1 Characterizing the Social Optimum

We assume a central planner that adopts a utilitarian social welfare function consisting of the discount sum of individual’s utilities. According to Blumkin and Sadka (2003), the social welfare function is defined by

(9) $$W = N[U_0(c_0,\ \pi_0,\ n,\ U_1) + \rho n U_1(c_1,\ \pi_1)].$$

Although the welfare of the children's generation is already incorporated into the social welfare function through the parent's utility, the central planner may also assign a positive weight to the children's welfare in itself. If this is the case, then $\rho > 0$. On the other hand, if the central planner counts the children's welfare only through the parent's utility, then $\rho = 0$. As a benchmark, we first assume $\rho = 0$, under which the social discount factor equals the parent's degree of altruism, and examine the case of $\rho > 0$ in Section 5.

Given k_0, A, β and α, the central planner chooses n and k so as to maximize (9). The first-order conditions are

(10) $$F^S(k,\ n) \equiv -nu_0'[Ak_0 - n(k+\beta)] + n[\delta(n)+\rho][Au_1'(Ak) - \alpha ANnV_1'(\alpha ANnk)] = 0,$$

(11) $$\begin{aligned} G^S(k,\ n) \equiv & -(k+\beta)u_0'[Ak_0 - n(k+\beta)] + [\delta(n) + n\delta'(n) + \rho][u_1(Ak) - V_1(\alpha ANnk)] \\ & - n[\delta(n)+\rho](\alpha ANk)V_1'(\alpha ANnk) = 0. \end{aligned}$$

We obtain the social optimum $(k^S,\ n^S)$ from (10) and (11). $G^S(k,\ n)$ can be defined as the social marginal net benefit (SMNB) of a child. (Similarly, $F^S(k,\ n)$ in (10) as SMNB of bequests.)

3.2 Comparing the Competitive Equilibrium to the Social Optimum

In our model, the parents do not take into account the effects of k and n on pollution *via* the production process. This implies that both childbearing and bequeathing to children have pollution externalities. Comparing (8) to

(11) with $\rho = 0$, it follows that the PMNB of n is greater than its SMNB by $n\delta(n)(\alpha ANk)V_1'$ given k. Similarly, a comparison between (7) and (10) with $\rho = 0$ indicates that the PMNB of k is greater than its SMNB by $n\delta(n)(\alpha ANn)V_1'$ given n. This does not imply, however, that k and n are determined higher in the competitive equilibrium than in the social optimum, because there exists an interaction between k and n. That is, an increase in bequests raises the marginal cost of having a child, and thus has a negative effect on the number of children. Therefore, if the level of capital accumulation in the competitive equilibrium is higher than the social optimal level, and this over-accumulation of capital lowers the PMNB of a child to a level below its SMNB, the number of children in the competitive equilibrium rather falls below that in the social optimum.[4]

Paying attention to the interaction of k and n, we now derive a condition for $n^* < n^S$. For this purpose, we define the following function:

(12) $$\hat{F}(k,\ n;\ \mu) \equiv F(k,\ n) - \mu\{n\delta(n)\alpha ANnV_1'(\alpha ANnk)\} = 0,$$

(13) $$\hat{G}(k,\ n;\ \mu) \equiv G(k,\ n) - \mu[n\delta(n)\alpha ANkV_1'(\alpha ANnk)] = 0.$$

Note that the competitive equilibrium (k^*, n^*) satisfies (12) and (13) when $\mu = 0$, whereas the social optimum $(k^S,\ n^S)$ satisfies them when $\mu = 1$. Furthermore, $\mu \in [0,\ 1]$ can be thought of as the parents' perceived rate of the effect of their behavior on pollution. Using this terminology, our model supposes the case where the parents' perceived rate is 0. On the other hand, if

4 Similarly, noting the impact of the number of children on the marginal cost of bequests, the relative magnitude of k^* and k^S is indeterminate.

it were 1, the competitive equilibrium would coincide with the social optimum.

Differentiating (12) and (13) yields[5]

$$\frac{dk}{d\mu}=\frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}F_n-G_n\right], \tag{14}$$

$$\frac{dn}{d\mu}=\frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}F_k+G_k\right], \tag{15}$$

where $F_k \equiv \partial F/\partial k\ (<0)$, $F_n \equiv \partial F/\partial n\ (<0)$, $G_k \equiv \partial G/\partial k\ (<0)$, $G_n \equiv \partial G/\partial n\ (<0)$, $\hat{F}_\mu \equiv \partial \hat{F}/\partial \mu\ (<0)$ and $D(\mu)\ (>0)$ is the determinant of the Jacobian.[6]

As shown in the Appendix, the sign of (15) is positive if

$$-n^*\delta(n^*)(\alpha ANk^*)V_1'(\alpha ANn^*k^*)+G_k(k^*,\ n^*)\left.\frac{\partial k}{\partial \mu}\right|_{\mu=0}>0, \tag{16}$$

where $(\partial k/\partial \mu)_{\mu=0}=-(\hat{F}_\mu/F_k)<0$. Since $dn/d\mu>0$ is a sufficient condition for $n^*<n^S$, we obtain the following proposition:

Proposition 1. If (16) is satisfied, then the number of children in the competitive equilibrium is smaller than that in the social optimum.

The intuition behind Proposition 1 is straightforward. The LHS of (16) represents the change in the marginal net benefit of a child when the parents' perceived rate rises from 0. The first term is the environmental effects of n which the parents do not take into account in calculating the PMNB of a child.

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5 See Appendix.

6 The proof of $D(\mu)>0$ is shown in the Appendix.

If they take these effects into account, the marginal net benefit of a child decreases. The second term is the effects through the change in capital (i.e., bequest). If the parents realize the environmental effects of k, they choose smaller amounts of bequests per child ($(\partial k / \partial \mu)_{\mu=0} < 0$). The decrease in bequests implies the decrease in the marginal cost of a child and the increase in the marginal net benefit of the child. Therefore, if the second term dominates the first term, the marginal net benefit of a child and thus the number of children increase. Figure 1 shows the SMNB of n and the PMNB of n when (16) is satisfied. Equation (16) implies that the PMNB is shifted upward by an increase in μ, and equivalently that the SMNB exceeds the PMNB, given $n = n^*$. Hence, the number of children determined in the competitive equilibrium is lower than the social optimum ($n^* < n^S$).

As to the comparison of the level of capital in the competitive equilibrium to its social optimal level, we obtain the following proposition:

Proposition 2. In the competitive equilibrium, if the number of children is insufficient, then capital is over-accumulated, relative to the social optimum.

Proof. See Appendix.

Together with Proposition 1, Proposition 2 implies that, if (16) is satisfied, $n^* < n^S$ and $k^* > k^S$ simultaneously holds.

The above result is in marked contrast with Jöst and Quaas (2009), in which the fertility rate in the competitive equilibrium is higher than that in the social optimum for two types of households: dynastic households and micro-households. The family (or household) considered here is basically the same as the dynastic household in Jöst and Quaas in that the parents decide their children's consumption, while our family consists only of two generations. In the case of dynastic households in Jöst and Quaas, while the cost of raising children depends on per-capita capital, there is no external effect from the firm's decision on capital and the equilibrium level of capital is socially optimal. On the other hand, in our model, the household's decision on bequests creates externality, which may lead to the higher cost of raising children and thus the lower fertility rate in comparison to the social optimum.

We next examine the relative magnitude of pollution between the competitive equilibrium and the social optimum. The result is summarized in the following proposition:

Proposition 3. The level of pollution is higher in the competitive equilibrium than in the social optimum.

Proof. See Appendix.

It should be noted that we have $\pi^* > \pi^S$ irrespective of the relative magnitude between n^* and n^S and between k^* and k^S. This is due to the external effects of childbearing and bequests on the environment. Furthermore, Propositions 1 and 3 imply that society may suffer from insufficient fertility and excess pollution at the same time. This result seems to be in line with a phenomenon prevailing in many developed countries.

3.3 Numerical Examples

In this sub-section, we consider numerical examples of our model to quantitatively assess the results obtained in the previous sub-section. For this purpose, we take a simple form of utility functions from goods consumption $u_0(c) = u_1(c) = c^{\sigma}$ and the disutility functions from pollution $V_0(\pi) = V_1(\pi) = B\pi^{\nu}/\nu$, where $0 < \sigma < 1$, $B > 0$, and $\nu > 1$. Following Becker and Barro (1988), the degree of altruism toward children is assumed to take a form with a constant elasticity with respect to the number of children, i.e., $\delta(n) = \xi n^{-\varepsilon}$, where $\xi > 0$ and $0 < \varepsilon < 1$.

Let us now determine the baseline values of parameters in the model. We normalize the population of parent's generation (N_0) to 1, and assume initial endowment of each parent (b_0) equal to 1. Thus, the total capital stock in the first period (K_0) equals 1. The productivity parameter (A) is set to 2.666. This value corresponds to the case where the interest rate on an annual basis is equal to 0.04, when one period is taken as 25 years (i.e., $2.666 \approx (1+0.04)^{25}$). The emission coefficient (α) is set to 0.15. We choose rearing cost per child (β) equal to 0.25, implying that the share of the child-rearing cost in income ($= n\beta/y_0$) is 12% in the competitive equilibrium. The values of other parameters are as follows: σ=0.6, B=1, ν=2, ξ=0.65, and ε=0.2. In this baseline case, the degree of altruism toward each child is about 0.6.

When each parameter is set to its baseline value, the competitive equilibrium is calculated as k^* =1.156 and n^* =1.271, while the social optimum is given by k^S =0.794 and n^S =1.424. Notice that, in the baseline case, the number of children in the competitive equilibrium is lower than the social optimum.

In the following, we perform sensitivity analysis by changing the value of key parameters in the model. In particular, we focus on the following parameters: the level parameter of the degree of altruism toward children (ξ), emission coefficient (α), and rearing cost per child (β). We change the value of each parameter in turn, and re-calculate k^*, n^*, π^*, k^S, n^S and π^S holding all other parameters fixed at their baseline values. The results are reported in Figures 2 – 4.

Figures 2(a) – 2(c) show the effects of changes in ξ. In Figure 2(a), the response of the level of capital in each case is presented. This shows that, while both k^* and k^S increase monotonically when ξ moves from 0.4 to 0.9, k^* increases more sharply than k^S. The response of the fertility rate to ξ is presented in Figure 2(b). This demonstrates that, over the same range of ξ, n^S increases monotonically, whereas n^* increases when ξ is relatively small, but decreases when ξ is relatively large. The number of children at a competitive equilibrium is too low as compared to the social optimum when the value of ξ is relatively large (i.e., $\xi \geq 0.5$), while it is too high when ξ is small (i.e., $\xi < 0.5$). Also, we can see that, as ξ becomes larger, the insufficiency in the number of children becomes greater.

The changes in the levels of pollution are plotted in Figure 2(c). π^S necessarily increases with ξ since both k^S and n^S increase as ξ increases. In our example, π^* also increases monotonically with ξ. As we saw above, while both k^* and n^* increase with ξ when ξ is relatively small, an increase in ξ raises k^* but lowers n^* when ξ is relatively large. However, the former effect dominates the latter in our example, and hence π^* increases with ξ, as shown in Figure 2(c). The figure also shows that, for all values of ξ considered here, the pollution is emitted too much in the competitive equilibrium as compared to the socially optimal level, and that the over-emission becomes greater as ξ becomes larger. By summing up the results obtained so far, we could say that, as the parents become more altruistic toward children, the competitive equilibrium becomes more inefficient.

Figures 3(a) – 3(c) show the effects of changes in α from 0.01 to 0.25 on k, n, and π respectively. From Figures 3(a) and 3(b), we can see the following. First, at both the competitive equilibrium and the social optimum, k monotonically increases and n monotonically decreases as α rises. Second, over the whole range of α, k^* is higher than k^S, and n^* is lower than n^S. Third, while k^* increases more sharply than k^S, n^* shows a sharper decrease than n^S. Hence, the gap between the competitive equilibrium and the social optimum widens when α rises. The changes in k

and n affect the pollution level. A rise in α may increase π through its positive effect on k, but may reduce π through its negative effect on n. In addition, a rise in α directly increases the emission of pollution for given k and n. In our example, the total effect of a rise in α on π is unambiguously positive, as shown in Figure 3(c). It can also be seen from the figure that the pollution is emitted more excessively in the competitive equilibrium as α rises.

Figures 4(a) – 4(c) show the effects of β on k, n, and π respectively. It can be seen from Figures 4(a) and 4(b) that an increase in β raises k and reduces n both in the competitive equilibrium and the socially optimum. We can also see that, over the whole range of β, the competitive equilibrium values of k and n are, respectively, too high and too low as compared to the social optimum. Figure 4(b) also shows that, as β increases, n^* decreases more sharply than n^S, and the gap between them becomes narrower. A change in β affects the pollution level through two channels. An increase in β may increase π through its positive effect on k, but may reduce π through its negative effect on n. In our example, the latter effect dominates the former in the competitive equilibrium, while these effects almost cancel out each other in the social optimum. Hence, an increase in β decreases π^* yet hardly affects π^S, as shown in Figure 4(c).

4. Optimal Policy

This section examines whether the social optimum can be decentralized.

In our model, since the parents fail to take into account the effects of production on pollution in choosing the number of children and the amount of bequests to each child, *laissez faire* leads both the fertility rate and per capita capital to become suboptimal. To control two variables, decentralization requires two policy instruments. Among many instruments the government can use, we consider taxes (or subsidies) on inheritance and taxes on childbearing (or child allowances), which would directly affect the decisions on fertility and bequests in the family.

4.1 Decentralizing the Social Optimum

The government budget is balanced by lump-sum transfers to private individuals in each period. We thus have

(17) $$nT = \theta,$$

(18) $$(1+r)b_1\tau = \eta,$$

where T is a tax per child imposed on the parents, τ is the tax rate on bequests to each child, θ is a lump-sum transfer to each parent, and η is a lump-sum transfer to each child.

The parent's utility function (1) is rewritten as

(19) $$\begin{aligned} U_0 = {} & u_0[(1+r)b_0 - n(b_1+\beta+T)+\theta] - V_0(\pi(Y_0)) \\ & + n\delta(n)\left\{u_1[(1-\tau)(1+r)b_1+\eta] - V_1(\pi(Y_1))\right\}. \end{aligned}$$

The competitive equilibrium in this case, $k = k^*(\tau,\ T)$ and $n = n^*(\tau,\ T))$, satisfies the following conditions:

(20) $$\begin{aligned} \tilde{F}(k,\ n;\ \tau,\ T) \equiv {} & -nu_0'[Ak_0 - n(k+\beta+T)+\theta] \\ & + n\delta(n)(1-\tau)Au_1'[(1-\tau)Ak+\eta] = 0, \end{aligned}$$

$$\text{(21)} \qquad \begin{aligned}\tilde{G}(k,\ n;\ \tau,\ T) \equiv & -(k+(\beta+T))u_0'[Ak_0 - n(k+\beta+T)+\theta] \\ & +(\delta(n)+n\delta'(n))\left\{u_1[(1-\tau)Ak+\eta]-V_1(\alpha ANnk)\right\}=0.\end{aligned}$$

If the government realizes the social optimum in a decentralized economy with $\tau = \tau^*$ and $T = T^*$, we have

$$\text{(22)} \qquad k^*(\tau^*,\ T^*) = k^S,$$

$$\text{(23)} \qquad n^*(\tau^*,\ T^*) = n^S.$$

Substituting (17), (18), (22) and (23) into (20) and (21) yields

$$\text{(24)} \qquad -n^S u_0'[Ak_0 - n^S(k^S+\beta)] + n^S\delta(n^S)(1-\tau^*)Au_1'(Ak^S) = 0,$$

$$\text{(25)} \qquad \begin{aligned}& -(k^S+\beta+T^*)u_0'[Ak_0 - n^S(k^S+\beta)] \\ & +(\delta(n^S)+n^S\delta'(n^S))[u_1(Ak^S)-V_1(\alpha ANn^S k^S)] = 0.\end{aligned}$$

Since (k^S, n^S) also satisfies (10) and (11) with $\rho = 0$, (24) and (10) with $k = k^S$ and $n = n^S$ imply

$$\text{(26)} \qquad \tau^* = \frac{\alpha Nn^S V_1'(\alpha ANn^S k^S)}{u_1'(Ak^S)} > 0.$$

Also, (25) and (11) with $k = k^S$ and $n = n^S$ imply

$$\text{(27)} \qquad T^* = \frac{n^S\delta(n^S)\alpha ANk^S V_1'(\alpha ANn^S k^S)}{u_0'[Ak_0 - n^S(k^S+\beta)]} > 0.$$

Hence we have the following proposition:

Proposition 4. If the social welfare function is given by (9) with $\rho = 0$, the social optimum can be decentralized with inheritance taxation and childbearing taxation that are defined in (26) and (27), respectively.

4.2 Implications of the Optimal Policy

Equation (27) implies that T^* is positive independent of the relative magnitude between n^* and n^S. (Similarly, (26) implies that τ^* is positive independent of the relative magnitude between k^* and k^S.) We now discuss why childbearing should be taxed to achieve social optimality, even when the number of children is insufficient relative to the social optimum.[7]

In Figure 5, lines F and G respectively represent (20) and (21) with $\tau = T = 0$ in the $(k,\ n)$ plane.[8] In this case, the number of children is too low and the level of bequests is too high at the equilibrium point E. Since an increase in τ shifts F to the left, and an increase in T shifts G downward,[9] these lines move to $\tilde{F}^*(=\tilde{F}(k,\ n;\ \tau^*,\ T^*))$ and $\tilde{G}^*(=\tilde{G}(k,\ n;\ \tau^*,\ T^*))$ if the government adopts $\tau = \tau^*(>0)$ and $T = T^*(>0)$. As a result, the social optimum S is achieved in the decentralized economy.

To explain why the government should tax childbearing although the number of children is insufficient in the initial equilibrium, we first suppose that the government uses only an inheritance tax τ as a policy tool. We see

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7 The reason inheritance should be taxed can be explained in a similar way.

8 Differentiating (20) and (21) with respect to k and n shows that both F and G slope downward, and F is steeper than G.

9 Differentiating (20) with respect to k, τ, T, θ and η, given n, and noting $Tdn + ndT = d\theta$ and $A(\tau dk + kd\tau) = d\eta$, which are derived from (17) and (18) respectively, yields $\partial k / \partial \tau = -(\tilde{F} / \tilde{F}_k) < 0$ and $\partial k / \partial T = 0$, where $\tilde{F}_k \equiv n^2 u_0'' + n\delta(n)(1-\tau)A^2 u_1'' < 0$ and $\tilde{F}_\tau \equiv -n\delta(n)Au_1' < 0$. Similarly, differentiating (21) with respect to n, τ, T, θ and η, given k, and noting $Tdn + ndT = d\theta$ and $A(\tau dk + kd\tau) = d\eta$ yields $\partial n / \partial \tau = 0$ and $\partial n / \partial T = -(\tilde{G}_T / \tilde{G}_n) < 0$, where $\tilde{G}_T \equiv -u_0' < 0$ and $\tilde{G}_n \equiv (k+\beta+T)(k+\beta)u_0'' + (2\delta'(n) + n\delta''(n))(u_1 - V_1) - (\delta(n) + n\delta'(n))\alpha NAkV_1' < 0$.

that an inheritance tax suffices to attain the optimal level of capital k^S as shown in Figure 5, in which a bequest tax of $\tau = \tau'$ alters the equilibrium to point D by shifting F to $\tilde{F}'$. However, the new equilibrium D is suboptimal because the number of children is too high relative to the social optimum ($n^*(\tau', 0) > n^S$). Once k is adjusted to its optimal level, the factor in the insufficiency of n disappears, and n exceeds its optimal level. In this stage, the government needs to tax childbearing to internalize a pollution externality children will create.

5. Alternative Social Discount Rate

Throughout the previous sections, we have maintained the assumption that the central planner counts the children's welfare only through the parent's welfare. In this section, we consider the case where the social discount factor differs from the parents' degree of altruism, namely, $\rho > 0$ in (9). The social optimum is characterized by (10) and (11) with $\rho > 0$.

In this case, the parents value the children's welfare less than the central planner, and the parents' behavior in terms of fertility and inheritance creates other types of externalities. As a result, while Proposition 1 would be maintained by slight modification of the sufficient condition, Proposition 2 is no longer valid. Whether k^* is higher or lower than k^S, the number of children can be too low due to the positive externality of childbearing that stems from the difference between the private and social welfare weights. Thus, $k^* > k^S$ is not a necessary condition for $n^* < n^S$.

Furthermore, we have policy implications different from those in the previous section. That is, Proposition 4 is not fully maintained in the sense that the social optimum can be still decentralized, but the optimal policy does not necessarily imply taxing both on childbearing and inheritance.

Equations (26) and (27) are reduced to

$$\tau^{**} = \frac{[\delta(n^S) + \rho]\alpha N n^S V_1'(\alpha A N n^S k^S) - \rho u_1'(Ak^S)}{\delta(n^S) u_1'(Ak^S)}, \tag{28}$$

$$T^{**} = \frac{n^S[\delta(n^S) + \rho]\alpha A N k^S V_1'(\alpha A N n^S k^S) - \rho[u_1(Ak^S) - V_1(\alpha A N n^S k^S)]}{u_0'[Ak_0 - n^S(k^S + \beta)]}. \tag{29}$$

The sign of τ^{**} and T^{**} may be positive or negative, depending on whether the pollution externalities, whose effects are captured by the first term in the numerator of (28) and (29), dominate or are dominated by the externalities arising from the parents' underestimation of the children's welfare, whose effects are captured by the second term in the numerator of (28) and (29). In contrast to the result under $\rho = 0$ (Proposition 4), therefore, child allowances and/or subsidies on bequests may be required to achieve optimality. It should be noted, however, that the sign of τ^{**} and T^{**} is not to be determined independent of the sign of the other. More specifically, we have that, if $\tau^{**} \le 0$, then $T^{**} < 0$, as shown in Appendix. This implies the following proposition:

Proposition 5. If the social welfare function is given by (9) with $\rho > 0$, a combination of taxes on childbearing and subsidies on inheritance cannot achieve the social optimum in a market economy.

We now examine relations numerically between the optimal policies (τ^{**}

and T^{**}) and the weight (ρ) on children's utilities in the social welfare function. Functional specifications and baseline values of the parameters are the same as in section 3.3. The optimal policies are then given as

$$\tau^{**} = \frac{0.06[0.65(n^S)^{-0.2} + \rho](n^S)^2(k^S) - 0.41(k^S)^{-0.4}\rho}{0.26(n^S)^{-0.2}(k^S)^{-0.4}}$$

$$T^{**} = \frac{n^S[0.65(n^S)^{-0.2} + \rho]0.16(n^S)(k^S)^2 - \rho[1.80(k^S)^{0.6} - 0.08(n^S)^2(k^S)^2]}{0.6[2.666 - (n^S)((k^S) + 0.25)]}$$

respectively, where k^S and n^S are obtained from the equations (10) and (11) for each ρ. Figure 6 shows the values of τ^{**} and T^{**} for ρ from 0 to 0.8. We can see from the figure that three cases appear depending on the value of ρ. If the weight (ρ) is moderate (i.e., $0.05 < \rho < 0.45$), then τ^{**} is positive and T^{**} is negative, i.e., inheritance should be taxed and child rearing should be subsidized. In contrast, when ρ is sufficiently small (i.e., $\rho \leq 0.05$), both τ^{**} and T^{**} are positive, whereas, when ρ is sufficiently large (i.e., $0.45 \leq \rho$), both τ^{**} and T^{**} are negative. That is, when the weight is sufficiently small, inheritance and child rearing should both be taxed, whereas, when the weight is sufficiently large, inheritance and child rearing should both be subsidized.

Many countries in the real world impose taxes on the bequests and support child rearing by giving parents child allowances, so the first case above (i.e., $\tau^{**} > 0$ and $T^{**} < 0$) seems to be prevalent in the real world. Meanwhile, the case of $\delta(n^S) + \rho = 1$ can be seen as typical because there are no a priori reasons for a social planner to differentiate the weights between individuals in the valuation of the utilities, although $\delta(n^S)+\rho$ is endogenous in our model. We can see that $\delta(n^S)+\rho \approx 1$ when $\rho = 0.3805$, and that $\tau^{**} > 0$ and $T^{**} < 0$ hold in this case. That is, although our numerical example may only be illustrative, we can demonstrate the prevalent case in the real world as a typical example. Also, in contrast to previous studies such as Harford (1998) and Jöst and Quaas (2009), in which taxes on the number of children or the household size are required to achieve optimality [10], this result suggests that child allowances constitute the optimal policy under a plausible assumption on the social welfare function.

6. Conclusion

Using an altruistic bequest model with endogenous fertility, in which both childbearing and bequests entail pollution externalities, we contrasted the fertility rate and the pollution level in the competitive equilibrium with those in the social optimum. It is shown that the fertility rate may be too low in the competitive equilibrium despite the negative externality created by childbearing, and, if this is the case, per capita capital over-accumulates. On the other hand, the level of pollution is unambiguously higher than the social optimum, whether the fertility rate (or per capita capital) is too high or too low.

Furthermore, we investigated what kind of policy is required to achieve social optimality. If the social discount factor for a child equals the private

[10] In our model where each generation lives only for one period, the number of children equals the household size.

degree of altruism, the government needs to tax both childbearing and inheritance so as to restore efficiency, even if fertility or capital accumulation falls short of the respective optimal level. On the other hand, if the social discount factor for a child is greater than the private degree of altruism, child allowances and/or subsidies to inheritance may be required to achieve optimality. It should be noted, however, that the optimal policy never involves a combination of taxes on childbearing and subsidies on inheritance. Our numerical examples suggest that, when the social discount factor is assigned equally among all individuals, inheritance taxes and child allowances, both of which have been adopted in many countries, can attain the social optimum.

Appendix

A.1. Derivation of $dk/d\mu$ and $dn/d\mu$:

Differentiating (25) and (26) with respect to k, n and μ yields

$$\begin{pmatrix} \hat{F}_k & \hat{F}_n \\ \hat{G}_k & \hat{G}_n \end{pmatrix} \begin{pmatrix} dk \\ dn \end{pmatrix} = -\begin{pmatrix} \hat{F}_\mu \\ \hat{G}_\mu \end{pmatrix} d\mu ,$$

where

$$\hat{F}_k = n^2 u_0'' + n\delta(n) A^2 u_1'' - \mu n\delta(n)(\alpha ANn)^2 V_1'' < 0,$$

$$\begin{aligned} \hat{F}_n &= n(k+\beta)u_0'' - u_0' + \left(\delta(n) + n\delta'(n)\right) Au_1' \\ &\quad - \mu[\left(2\delta(n) + n\delta'(n)\right)(\alpha ANn)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''] \\ &= n(k+\beta)u_0'' + n\delta'(n)Au_1' \\ &\quad - \mu[\left(\delta(n) + n\delta'(n)\right)(\alpha ANn)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''] < 0 \end{aligned}$$

$$(\because u_0' = \delta(n)Au_1' - \mu\delta(n)(\alpha ANn)V_1'),$$

$$\begin{aligned} \hat{G}_k &= n(k+\beta)u_0'' - u_0' + (\delta(n) + n\delta'(n))[Au_1' - (\alpha ANn)V_1'] \\ &\quad - \mu\left[n\delta(n)(\alpha AN)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''\right] \\ &= n(k+\beta)u_0'' + n\delta'(n)Au_1' - (\delta(n) + n\delta'(n))(\alpha ANn)V_1' \\ &\quad - \mu\left[n\delta(n)(\alpha AN)^2 nkV_1''\right] < 0 \end{aligned}$$

$$(\because u_0' = \delta(n)Au_1' - \mu\delta(n)(\alpha ANn)V_1'),$$

$$\begin{aligned} \hat{G}_n &= (k+\beta)^2 u_0'' + (2\delta'(n) + n\delta''(n))(u_1 - V_1) - (\delta(n) + n\delta'(n))(\alpha ANk)V_1' \\ &\quad - \mu\left[(\delta(n) + n\delta'(n))(\alpha ANk)V_1' + n\delta(n)(\alpha ANk)^2 V_1''\right] < 0, \end{aligned}$$

$$\hat{F}_\mu = -n\delta(n)(\alpha ANn)V_1' < 0 ,$$

$$\hat{G}_\mu = -n\delta(n)(\alpha ANk)V_1' < 0 .$$

Noting that $\hat{G}_\mu = \hat{F}_\mu k/n$, we have

$$\frac{dk}{d\mu}=\frac{1}{D(\mu)}\left[-\hat{F}_\mu\hat{G}_n+\hat{G}_\mu\hat{F}_n\right]=\frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}\hat{F}_n-\hat{G}_n\right]=\frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}F_n-G_n\right],$$

$$\frac{dn}{d\mu}=\frac{1}{D(\mu)}\left[-\hat{G}_\mu\hat{F}_k+\hat{F}_\mu\hat{G}_k\right]=\frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}\hat{F}_k+\hat{G}_k\right]=\frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}F_k+G_k\right].$$

A.2. Proof of $D(\mu)>0$

We have $D(\mu)=D(0)+\mu\varphi\ (0\le\mu\le1)$, where $\varphi=F_k\hat{G}_{n\theta}+G_n\hat{F}_{k\theta}-F_n\hat{G}_{k\theta}-G_k\hat{F}_{n\theta}$, $\hat{G}_{n\mu}\equiv\partial\hat{G}_n/\partial\mu=-[(\delta(n)+n\delta'(n))(\alpha ANk)V_1'+n\delta(n)(\alpha ANk)^2V_1'']$, $\hat{F}_{k\mu}\equiv\partial\hat{F}_k/\partial\mu=-n\delta(n)(\alpha ANn)^2V_1''$, $\hat{G}_{k\mu}\equiv\partial\hat{G}_k/\partial\mu=-n\delta(n)(\alpha AN)^2nkV_1''$ and $\hat{F}_{n\mu}\equiv\partial\hat{F}_n/\partial\mu=-[\left(\delta(n)+n\delta'(n)\right)(\alpha ANn)V_1'+n\delta(n)(\alpha AN)^2nkV_1'']$. The second-order condition for parental utility maximization and the condition for stability of the competitive equilibrium imply $D(0)>0$, and the second-order condition for social welfare maximization implies $D(1)>0$. Since φ does not depend on μ, we have $D(\mu)>0\forall\mu$. □

A.3. Proof of Proposition 1

Noting that $\hat{G}_\mu=(k/n)\hat{F}_\mu$, we rewrite (15) as

(A1) $$\frac{dn}{d\mu}=\frac{F_k}{D(\mu)}\left[-\hat{G}_\mu+\frac{\hat{F}_\mu}{F_k}G_k\right].$$

Differentiating (12) with respect to k and μ and substituting $\mu=0$ into the resulting equation yields

(A2) $$\left.\frac{\partial k}{\partial \mu}\right|_{\mu=0} = -\frac{\hat{F}_\mu}{F_k}$$

From (A1), (A2) and $\hat{G}_\mu = -n\delta(n)(\alpha ANk)V'$, we have

(A3) $$\left.\frac{dn}{d\mu}\right|_{\mu=0} = -\frac{F_k(k^*,\, n^*)}{D(\mu)}\left[-n^*\delta(n^*)(\alpha ANk^*)V_1'(\alpha ANn^*k^*) + G_k(k^*,\, n^*)\left.\frac{\partial k}{\partial \mu}\right|_{\mu=0}\right].$$

Noting that the sign of $dn/d\mu$ does not depend on μ, (A3) implies that, if (16) is satisfied, then $dn/d\mu > 0 \forall \mu$. □

A.4. Proof of Proposition 2

Defining $k^*(n^S)$ as k that satisfies $F(k,\, n^S) = 0$, (7) and (10) imply that $k^*(n^S) > k^S$. Differentiating (7) with respect to k and n yields $\partial k / \partial n = -F_n / F_k < 0$. Hence, if $n^* \le n^S$, then $k^* \ge k^*(n^S) > k^S$.

A.5. Proof of Proposition 3

Differentiating $\pi_1 = \alpha ANnk$ with respect to μ and substituting (14) and (15) yields

(A4) $$\frac{d\pi}{d\mu} = \frac{\alpha AN\hat{F}_\mu}{D(\mu)}\left[n\left(\frac{k}{n}F_n - G_n\right) + k\left(-\frac{k}{n}F_k + G_k\right)\right].$$

Furthermore, subsituting $G_k = F_n - (\delta(n) + n\delta'(n))\alpha ANnV_1'$ and $G_n = U_{nn} - (\delta(n) + n\delta'(n))\alpha ANkV_1'$ (where $U_{nn} \equiv \partial^2 U_0 / \partial n^2$) into (A4) yields

$$\frac{d\pi}{d\mu} = \frac{-\alpha ANn\hat{F}_\mu}{D(\mu)}\left[\left(\frac{k}{n}\right)^2 F_k - 2\frac{k}{n}F_n + U_{nn}\right]$$

$$= \frac{-\alpha ANn\hat{F}_\mu}{D(\mu)F_k}\left[(\frac{k}{n}F_k - F_n)^2 + F_k U_{nn} - (F_n)^2\right].$$

We have $F_k U_{nn} - (F_n)^2 > 0$ from the second-order conditions for the parent's utility maximization, and hence $d\pi / d\mu < 0$. □

A.6. Proof of Proposition 5

Using (10) and (11), we rewrite (28) and (29), respectively, as follows:

(A5) $$\hat{\tau}^* = \frac{[\delta(n^S)+\rho]\delta(n^S)\alpha ANn^S k^S V_1' - \rho k^S u_0'}{Ak^S[\delta(n^S)+\rho]\delta(n^S)u_1'},$$

(A6) $$\hat{T}^* = \frac{[\delta(n^S)+\rho][\delta(n^S)+n^S\delta'(n^S)]\alpha ANn^S k^S V_1' - \rho(k^S+\beta)u_0'}{[\delta(n^S)+\rho+n^S\delta'(n^S)]u_0'}.$$

Subtracting the numerator of (A6) from that of (A5) yields

$$-n^S\delta'(n^S)[\delta(n^S)+\rho]\alpha ANn^S k^S V_1' + \rho\beta u_0' > 0\,.$$

Since the denominators of (A6) and (A5) are both positive, we have that, if $\hat{\tau}^* \le 0$, then $\hat{T}^* < 0$. □

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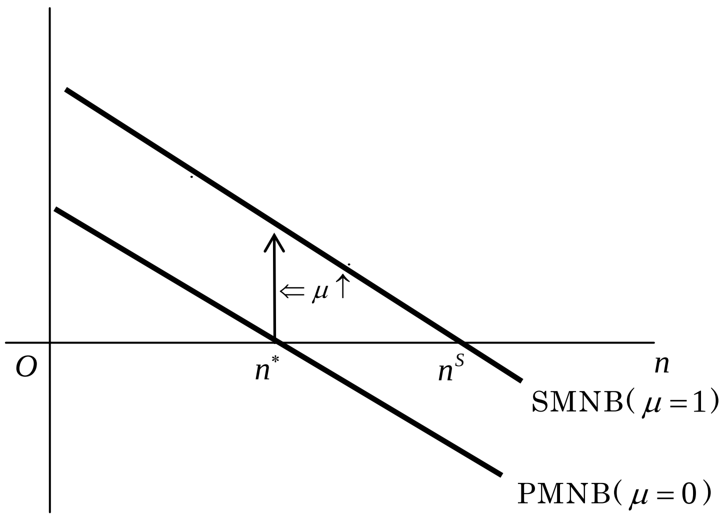

Figure 1: Number of children: competitive equilibrium and social optimum

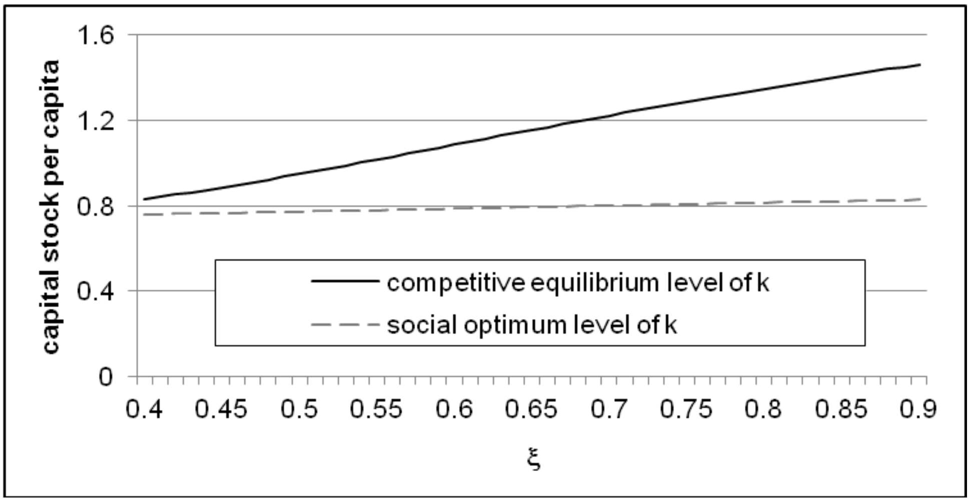

Figure 2(a): Sensitivity of capital stock per capita to ξ

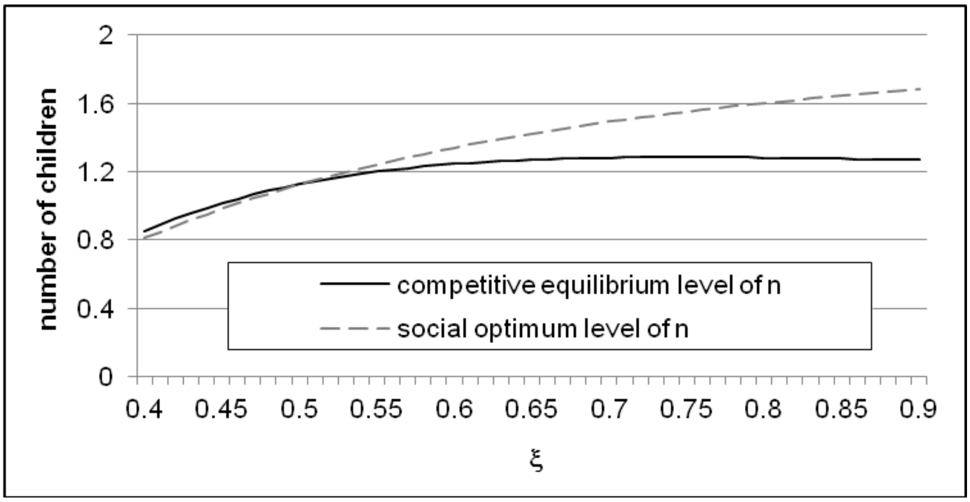

Figure 2(b): Sensitivity of number of children to ξ

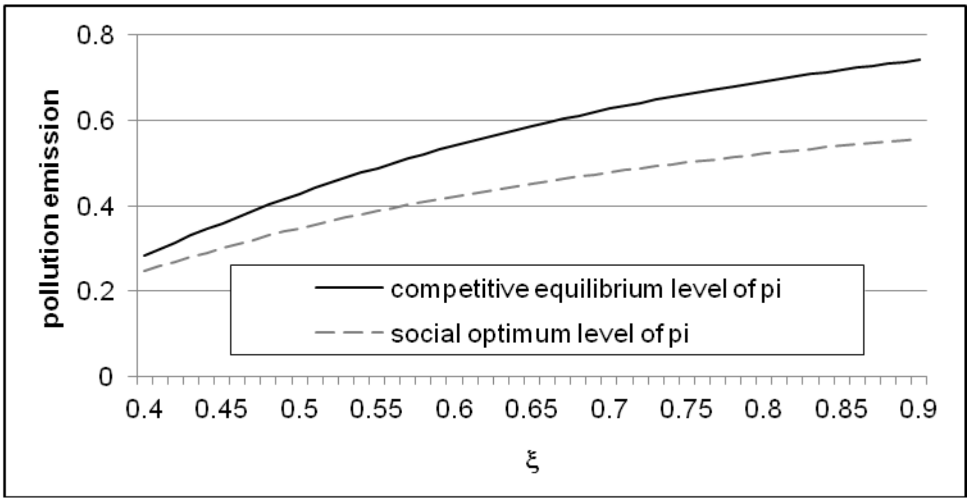

Figure 2(c): Sensitivity of pollution emission to ξ

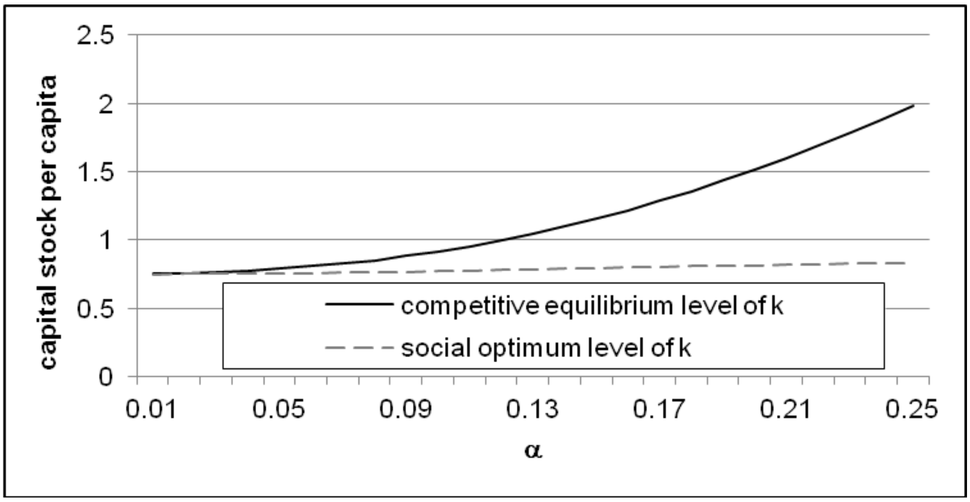

Figure 3(a): Sensitivity of capital stock per capita to α

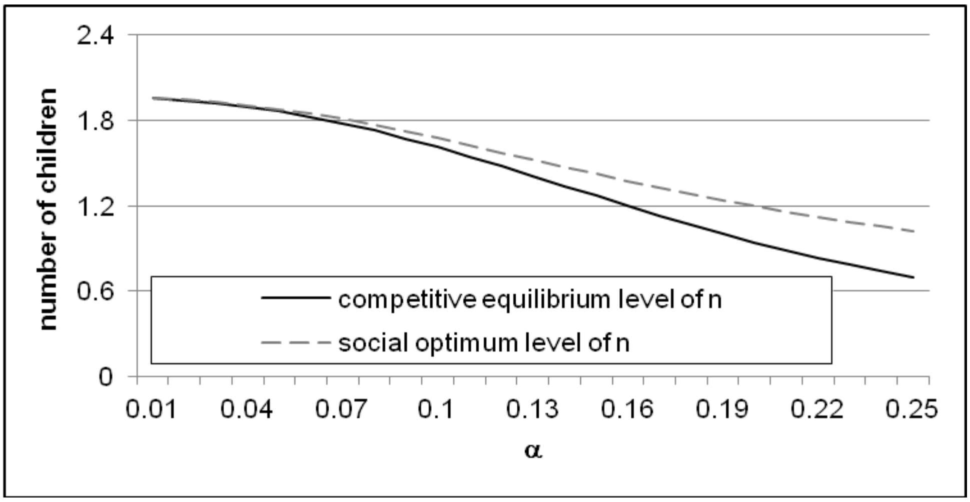

Figure 3(b): Sensitivity of number of children to α

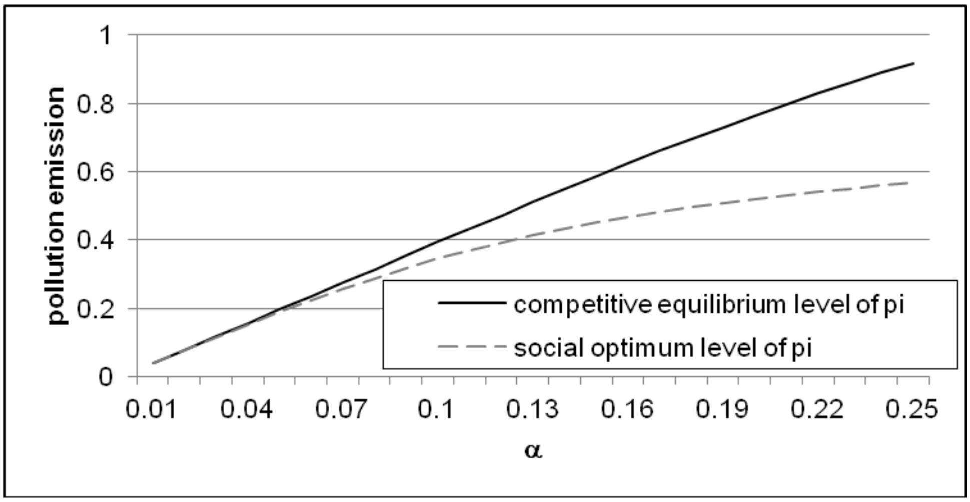

Figure 3(c): Sensitivity of pollution emission to α

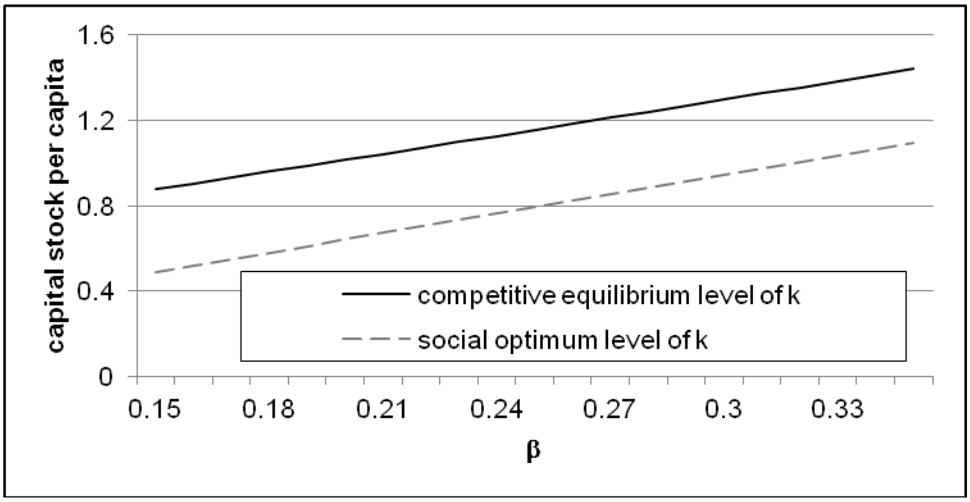

Figure 4(a): Sensitivity of capital stock per capita to β

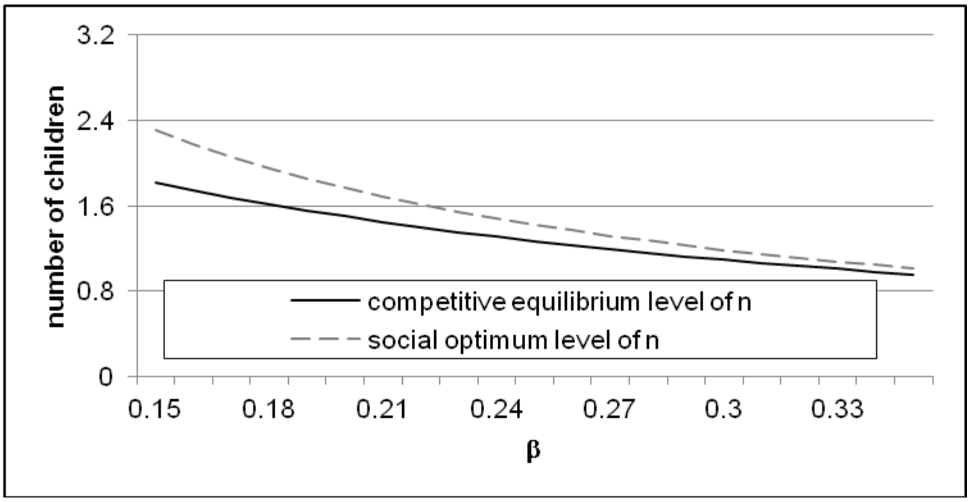

Figure 4(b): Sensitivity of number of children to β

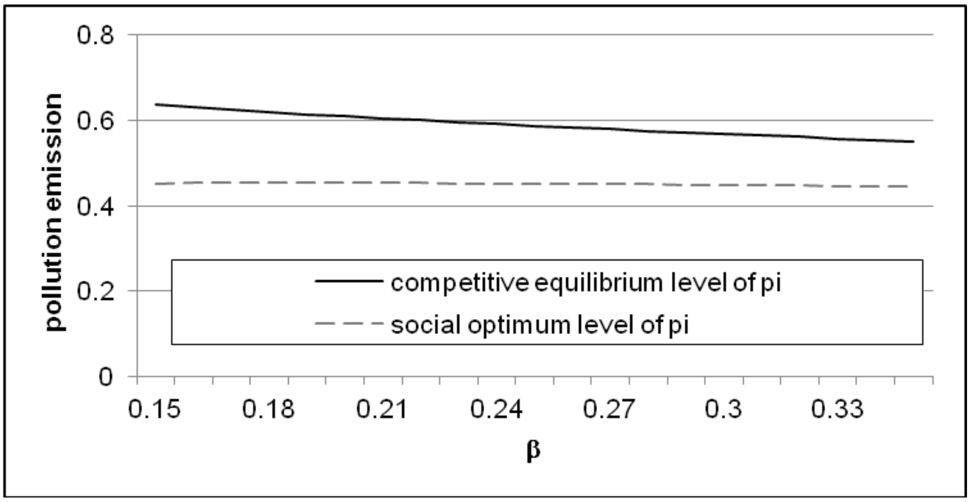

Figure 4(c): Sensitivity of pollution emission to β

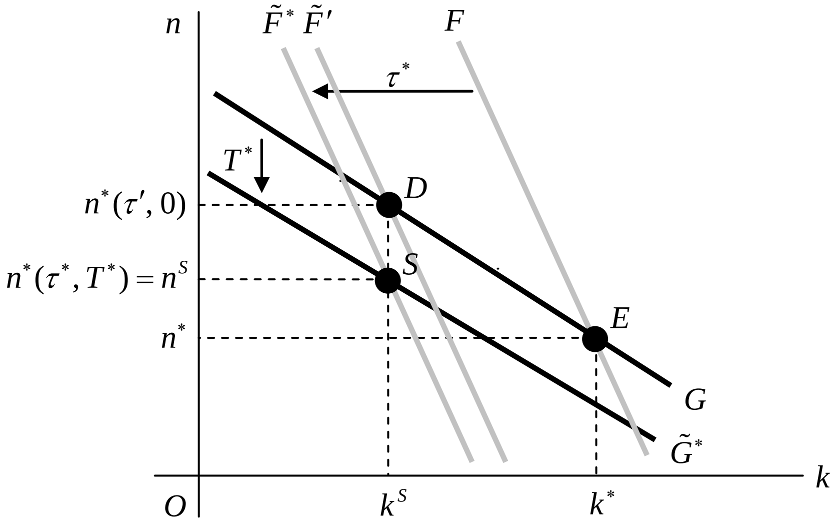

Figure 5: Optimal policies when $\rho = 0$

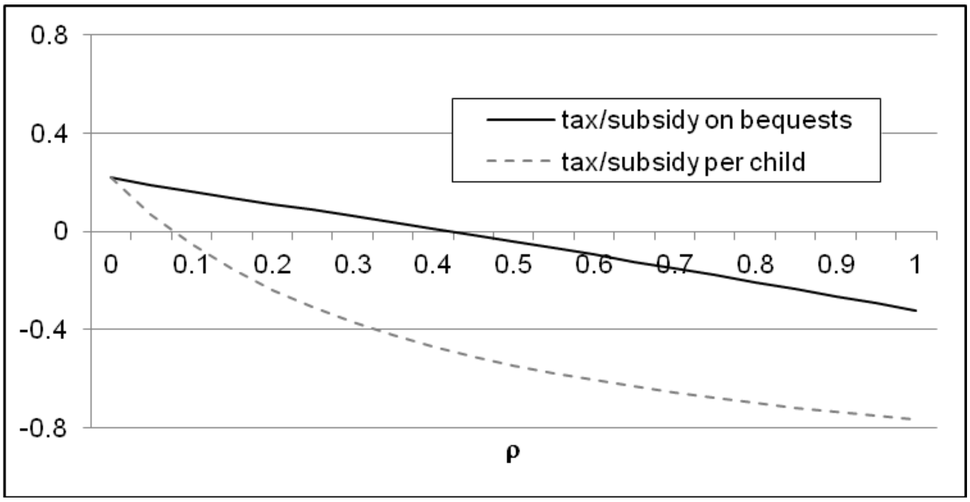

Figure 6: Optimal policies for various values of ρ