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illegal immigration

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# Optimal restriction policies for two co-existing types of illegal immigration

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## **Abstract**

We consider two possible cases of illegal immigrants to a developed country based on their expected lifetime income: high-risk entrance with low-risk of detection during their stay (type F migration) and low-risk entrance with high risk of detection (type S migration). We find that if medium productivity workers migrate illegally as type S immigrants, and if lower productivity workers migrate illegally as type F, internal investigation policies for type S immigrants will contribute to enhance the average productivity of illegal immigrants. On the other hand, if medium productivity workers migrate illegally as type F while lower productivity workers migrate illegally as type S, border control policies for type F immigrants will contribute to enhance the average productivity of illegal immigrants. Moreover, we find that the expansion of legal immigration (legalization of high productivity illegal immigrants) by the host country reduces the total number of illegal immigrants and enhances the average productivity of total immigrants in the two cases that were examined in this paper.

**Keywords:** illegal immigration, border control, internal investigation

**JEL code:** F22, J61

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## 1. Introduction

According to the Ministry of Justice, the number of illegal residents in Japan was about 63,000 as of January 1, 2016, with the most coming from South Korea (around 13,000), followed by China.<sup>1</sup> Around two-thirds of those residents are people who entered Japan as short-period visitors such as tourists and then overstayed. Ten percent of them entered as technical interns who were permitted to stay in Japan for a limited period to acquire skills to contribute to their home countries' development. 5.5% of them are non-Japanese spouses of Japanese nationals. Obviously international marriage itself is a legal residence status, but some foreigners enter Japan by means of a fake marriage. In addition, around 5.5% of them entered Japan as students, then neglected their studies and engaged in part-time jobs beyond the permitted number of hours, and continued to stay in Japan illegally after their visa status expired.

We classify illegal residents in developed countries into two categories. Based on the expected lifetime income, some illegal immigrants to developed countries prefer a high-risk and high-cost entrance with a low-risk of detection during their stay. Immigrants by fake marriage or disguised political refugees are included in this category. They spend a considerable amount of money to obtain fake documents that need to be submitted at the border to the immigration bureau. Even when the quality of fake documentation is high, some of them are detected by border control, and in those cases, they must pay penalty charges and return to their home country immediately. But if they successfully pass border control and enter as legal permanent residents, their risk of being detected are quite low during their stay since opportunities to re-evaluate their status are quite limited. On the other hand, some illegal immigrants prefer a low-risk and low-cost entrance with a high risk of detection during their stay. Most overstaying residents are included in this category. As tourists, technical interns or students, their entrances are costless and completely legal, but after their visas expire, they face the risk of detection by internal investigation actions taken by the host country government.

There are numerous studies about illegal immigration. Ethier (1986) is the pioneering theoretical study that introduced two common restriction policies to address illegal immigration, border control (border enforcement), and internal investigation (internal enforcement). Ethier concludes that by border control (and also by internal

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<sup>1</sup> Not all illegal residents are captured in government data. For example, it is impossible to count the number of fake permanent residence visa holders (including fake refugees) unless their false status are uncovered.

investigation, when employers can assess the immigration status of their employees), it is not possible to simultaneously accomplish two political targets (i.e., reduction of the volume of illegal immigrants and the improvement of the productivity of unskilled workers). But he suggests that the combination of the two policies could reduce the cost of restriction. We note that Ethier refers to two different types of restriction policies but does not distinguish between different groups of immigrants who are targeted by these two policies. Different policies are considered to be effective to all illegal immigrants.<sup>2</sup> We also note that previous studies sometimes assume that the host country government adopts only one of the restriction policies. For example, Bond and Chen (1986) and Yoshida (1993) take only the internal investigation policy into consideration and focus on its effect on the economy of the host country and the global economy. Using an efficiency model, Carter (1999) also studies the economic effects of illegal immigration referring only to the internal investigation policy. Djajic (1987) is an extended dynamic model of Harris and Todaro (1970), and also implicitly considers the existence of internal enforcement.<sup>3</sup> As an example of a theoretical analysis with border control policy, we refer to Yoshida (1998), which re-examines the Bond and Chen (1987) model using a different immigration policy. Based on this, we assert that an important aspect has been ignored. Different policies may be considered effective to different types of illegal immigrants, that is, border control may reduce only some types of illegal immigrants such as those with fake visas, and internal investigation can mainly detect illegal immigrants in another category such as overstaying residents. However, past studies have not focused on this aspect.

On the other hand, the worker's optimal choice of whether to be a legal migrant, an illegal migrant, or continue to stay home (in a developing country), was analyzed by Kondoh (2000), considering the difference in the potential ability of each worker and the skill formation period required to be a legal skill holder as seen in Djajic (1989). But there exist no studies about the choice of a potential illegal migrant, whether to enter with high risk and high cost but stay with low risk, or to enter with low risk and low cost but stay with high risk. To address the former type of illegal immigrants, the effective restriction policy should be border control, while for the latter type border control is meaningless, but internal investigation could successfully contribute to their detection.

In this study, we attribute the reason of this different behavior of illegal

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<sup>2</sup> One good example is Ethier (1986), in which he considers the possible substitution between the two restriction policies.

<sup>3</sup> Djajic (1997) also studies illegal migration, but in this study the difference between illegal and legal immigrants is that the former can only be employed in the informal sector.

immigrants to the individuals' skill diversification. We focus on the economy of the host country and analyze optimal policies to reduce the total number of illegal immigrants and/or enhance the average quality of immigrants. In addition, we pay particular attention to the effects of legalization of illegal immigrants with relatively high productivity.

In Section 2, we present our model. Section 3 is devoted to the analysis, and concluding remarks are presented in Section 4.

## 2. The Model

Let us consider a world with two countries, and to specify the analysis of the effects of international migration, let us assume that both countries produce the same one good and thus there is no international trade as in MacDougall (1960). The primary factors of production are capital and labor. We assume that the quality of each unit of labor is unique. There is no difference in the production technology or in labor endowment (in terms of absolute quantity and quality of workers) between the two countries. Due to the difference in the capital endowment, one country is relatively capital abundant (let us call it "Country R"), while the other is labor abundant ("Country Q"). Under the assumption of full employment and perfect competition of factor markets, because of the wage gap, there exists motivation for international migration from Country Q to R. On the other hand, we assume no international capital movement.<sup>4</sup>

In this study, we consider the diversification of skill in each worker of Country Q, which directly reflects his/her productivity and wage rate. Individuals differ in productivity,  $\theta$ , and they are uniformly distributed between  $[0,1]$ . Following Katz and Stark (1984) and Stark (1991), we express the wage rate of each worker in Countries R and Q as a function of his/her productivity,

$$w_R(\theta) = r_0 + r\theta, \quad (1)$$

$$w_Q(\theta) = q_0 + q\theta, \quad (2)$$

where  $w_R$  and  $w_Q$  are, respectively, the wage rates of an individual in Countries R and

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<sup>4</sup> We may consider that capital is a kind of immobile factor like land, or that by political reasons the government of Country R strictly restricts capital outflow.

Q.  $r_0$  and  $q_0$  are, respectively, the basic wage rate of a worker in Countries R and Q whose productivity is at the lowest level. Each worker's wage rate increases depending on his/her productivity by rates  $r$  and  $q$ , respectively in Countries R and Q. Note that from the difference in capital endowment,  $w_R > w_Q$  for all  $\theta$ .

As mentioned in the Introduction section, each worker intends to maximize his/her expected lifetime income. As a result, some workers prefer to adopt a low-risk and low-cost entrance with a high risk of detection. Let us call them type S migrants. They enter as tourists, technical interns, or language school students and overstay after the expiration of their visas. We assume that there is no special cost or inspection associated with crossing the border. Because they do not have work permits, they live in constant fear of being discovered. Employers must pay penalty charges in case of detection. Thus following Yoshida (1993), immigrants' wage rates should be discounted to account for this probability. We express the expected income of a type S migrant,  $w_S$ , with their innate productivity  $\theta$  as follows,

$$w_S(\theta) = \gamma k(w_R(\theta) - p\varepsilon) + (T - \gamma(p))w_Q(\theta), \quad (3)$$

where  $p$  denotes the probability of detection in each period during their illegal stay,  $\varepsilon$  denotes the penalty charge in case of detection that employers must pay, and  $T$  denotes his/her survival period.<sup>5</sup> Additionally,  $\gamma$  denotes the expected or average period of illegal stay, and we reasonably assume it is a decreasing function of  $p$ , that is  $\gamma = \gamma(p)$  and  $\gamma' < 0$ .

A worker prefers (does not prefer) to be a type S illegal migrant when

$$f(\theta) \equiv w_S(\theta) - w_Q(\theta) > (<) 0, \quad (4)$$

is satisfied. Let  $\theta_{Fi}$  denote the marginal productivity level that satisfies  $f(\theta_{Fi}) = 0$ , where  $i = A, B$ , which are two possible cases to be explained later.

On the other hand, some workers in Country Q prefer to migrate to a developed country by choosing a high-risk and high-cost entrance with a low-risk of detection during their stay. Let us call them type F migrants. They disguise themselves as refugees

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<sup>5</sup> To simplify our analysis, we ignore the ordinary discount rate caused by time preference.

or legal skilled workers. Unlike type S immigrants, we assume it is necessary for these migrants to pay a special cost to obtain fake visas. Also, if their illegal status is discovered, they must pay penalty charges themselves. However, once they successfully pass border control, then they face a fairly low risk of detection during their stay. We express the expected income of a type F migrant,  $w_F$ , with innate productivity  $\theta$  as follows,

$$w_F(\theta) = T[\eta k w_R(\theta) + (1 - \eta) w_Q(\theta)] - \mu, \quad (5)$$

where  $\eta$  denotes the probability of success in crossing the border,  $\mu$  denotes the necessary cost to obtain the fake visa, and  $k (< 1)$  denotes the discount factor caused by being away from his family. A worker prefers (does not prefer) to migrate from Country Q to R when

$$g(\theta) \equiv w_F(\theta) - w_Q(\theta) > (<) 0, \quad (6)$$

is satisfied. Let  $\theta_{Gi}$  denote the marginal productivity level that satisfies  $g(\theta_{Gi}) = 0$ .

We now consider two possible cases in which both types of illegal migration co-exist. The first case, Case A, requires the following three conditions to be met:

$$0 > g' > f', \quad (7-1)$$

$$f(0) > g(0), \quad (7-2)$$

$$\theta_{GA} > \theta_{FA}. \quad (7-3)$$

We note that (7-1) implies  $kr_0 > q_0$ ,  $kr > q$ , and  $\gamma(p) > \eta T$ . Under the above conditions, individuals whose productivities fall in the range  $[0, \theta_A]$  prefer to be type S migrants, while those in the range  $[\theta_A, \theta_{GA}]$  prefer to be type F migrants, where  $\theta_A$  is the crucial productivity level that satisfies  $f(\theta_A) = g(\theta_A)$ . In Case A, relatively low

productivity workers migrate by way of low-risk and low-cost entrance with high risk of detection. Medium level workers in Country Q, on the other hand, prefer to migrate by way of high-risk and high-cost entrance with low risk of detection.

In Case A, the real number of type S illegal migrants from Country Q to R can be expressed as,

$$\gamma(p)\theta_A, \quad (8)$$

while the efficiency unit of the inflow of type S illegal migrants to Country R considering their productivity can be expressed as,

$$L_{SA}^R = \gamma(p)r \int_0^{\theta_A} \theta d\theta = \gamma(p)\frac{r}{2}\theta_A^2, \quad (9)$$

and the efficiency unit of the outflow of type S illegal migrants from Country Q can be expressed as,

$$L_{SA}^Q = \gamma(p)q \int_0^{\theta_A} \theta d\theta = \gamma(p)\frac{q}{2}\theta_A^2. \quad (10)$$

Similarly, we express the real number, efficiency unit of the inflow, and the outflow of type F migrants as follows,

$$\eta T(\theta_{GA} - \theta_A), \quad (11)$$

$$L_{FA}^R = \eta T r \int_{\theta_A}^{\theta_{GA}} \theta d\theta = \eta T \frac{r}{2}(\theta_{GA}^2 - \theta_A^2), \quad (12)$$

$$L_{FA}^Q = \eta T q \int_{\theta_A}^{\theta_{GA}} \theta d\theta = \eta T \frac{q}{2}(\theta_{GA}^2 - \theta_A^2). \quad (13)$$

The basic wage rate in Countries R and Q after migration is,

$$r_0 = r_0(L_{SA}^R + L_{FA}^R) = r_0\left(\gamma(p)\frac{r}{2}\theta_A^2 + \eta T \frac{r}{2}(\theta_{FA}^2 - \theta_A^2)\right), \quad (14)$$

$$q_0 = q_0(L_{SA}^O + L_{FA}^O) = q_0(\gamma(p)\frac{q}{2}\theta_A^2 + \eta T\frac{q}{2}(\theta_{FA}^2 - \theta_A^2)), \quad (15)$$

where  $r_0'(L_{SA}^R + L_{FA}^R) < 0$  and  $q_0'(L_{SA}^O + L_{FA}^O) > 0$ .

Wage rates of an individual in Countries R and Q, respectively, whose productivity is  $\theta$  can be expressed as in (1) and (2), and noting that each worker's wage rate represents his/her productivity, which remains unchanged before and after migration, we note that  $r_0 + r\theta/r_0$  is constant and thus obtain,

$$dr = (r^*/r_0^*)dr_0, \quad (16-1)$$

where  $r^*$  and  $r_0^*$  are, respectively, the basic wage rate and its rate of increase depending on the worker's productivity without labor mobility in Country R. Similarly, we also have

$$dq = (q^*/q_0^*)dq_0, \quad (16-2)$$

where  $r^*$  and  $r_0^*$  are, respectively, the basic wage rate and its rate of increase depending on the worker's productivity without labor mobility in Country Q.

Next, let us consider another possible case of co-existence of the two types of illegal immigrants. The second case, Case B, requires satisfaction of the following three conditions:

$$0 > f' > g', \quad (17-1)$$

$$f(0) < g(0), \quad (17-2)$$

$$\theta_{FB} > \theta_{GB}. \quad (17-3)$$

We note that (17-1) implies  $kr_0 > q_0$ ,  $kr > q$ , and  $\gamma(p) < \eta T$ . Under the above conditions, individuals whose productivities fall in the range  $[0, \theta_B]$  prefer to be type F migrants, while those in the range  $[\theta_B, \theta_{FB}]$  prefer to be type S migrants, where  $\theta_B$  is the crucial productivity level satisfying  $f(\theta_B) = g(\theta_B)$ . In Case B, relatively low

productivity workers migrate by way of high-risk and high-cost entrance with low risk of detection. Medium level workers in Country Q, on the other hand, prefer to migrate by way of low-risk and low-cost entrance with high risk of detection

In Case B, the real number of type S illegal migrants from Country Q to Country R can be expressed as,

$$\eta T \theta_B, \quad (18)$$

while the efficiency unit of the inflow of type S illegal migrants to Country R considering their productivity can be expressed as,

$$L_{FB}^R = \eta T r \int_0^{\theta_B} \theta d\theta = \eta T \frac{r}{2} \theta_B^2, \quad (19)$$

and the efficiency unit of the outflow of type S illegal migrants from Country Q can be expressed as,

$$L_{FB}^Q = \eta T q \int_0^{\theta_B} \theta d\theta = \eta T \frac{q}{2} \theta_B^2. \quad (20)$$

Similarly, we express the real number, efficiency unit of the inflow, and the outflow of type F migrants as follows,

$$\gamma(p)(\theta_{FB} - \theta_B), \quad (21)$$

$$L_{SB}^R = \gamma(p)r \int_{\theta_B}^{\theta_{FB}} \theta d\theta = \gamma(p) \frac{r}{2} (\theta_{FB}^2 - \theta_B^2), \quad (22)$$

$$L_{SB}^Q = \gamma(p)q \int_{\theta_B}^{\theta_{FB}} \theta d\theta = \gamma(p) \frac{q}{2} (\theta_{FB}^2 - \theta_B^2). \quad (23)$$

The basic wage rate in Countries R and Q after migration is,

$$r_0 = r_0 (L_{FB}^R + L_{SB}^R) = r_0 \left( \eta T \frac{r}{2} \theta_B^2 + \gamma(p) \frac{r}{2} (\theta_{SB}^2 - \theta_B^2) \right), \quad (24)$$

$$q_0 = q_0(L_{FB}^O + L_{SB}^O) = q_0(\eta T \frac{q}{2} \theta_B^2 + \gamma(p) \frac{q}{2} (\theta_{SB}^2 - \theta_B^2)), \quad (25)$$

where  $r_0'(L_{FB}^R + L_{SB}^R) < 0$  and  $q_0'(L_{FB}^O + L_{SB}^O) > 0$ . Clearly in Case B, (16) is also valid.

### 3. Analysis

Generally speaking, labor inflow causes positive effects on the economy of the host (recipient) country. However, this fact may not be appreciated due to security concerns by increased crime, or to protect those whose jobs may be taken away by the immigrants. Thus, we assume that Country R, the host country of illegal immigration, intends to realize two policy targets to address illegal immigrants, that is, to reduce their total number and/or to enhance the average productivity of these immigrants.

In Case A, from the definitions of  $\theta_A$  and  $\theta_{GA}$ , we obtain the following two equations,

$$\eta T(kr - q)\theta_{GA} = -\eta T(kr_0 - q_0) + \mu, \quad (26)$$

$$(kr - q)[\gamma(p) - \eta T]\theta_A = -[\gamma(p) - \eta T](kr_0 - q_0) + \gamma(p)kp\varepsilon - \mu. \quad (27)$$

In Case A, recalling (16-1) and (16-2), we have four equations, (14), (15), (26), and (27), from which four endogenous variables,  $\theta_A, \theta_{GA}, r_0$  and  $q_0$  will be determined.

We note again that from (7-1),  $\gamma(p) > \eta T$  is satisfied in Case A, which implies that the expected duration of a type S immigrant in the host country is longer than that of a type F immigrant. Thus, a decrease in both  $\theta_A$  and  $\theta_{GA}$  would be sufficient to reduce the total number of illegal immigrants, and increasing  $\theta_{GA}$  with decreasing  $\theta_A$  should be sufficient to enhance the average quality of illegal immigrants staying in Country R at any given point.

As mentioned in the Introduction section, two possible restriction policies are

available for Country R, border control and internal investigation. Unlike Ethier (1986) and most other studies, in our model, the targets of these two policies are completely separated. Stricter border control implies stricter visa inspection at the border, which will directly reduce type F immigrants' probability of success in crossing the border, while stricter internal investigation will likely reduce the expected duration of type S immigrants in the host country.

To restrict the inflow of type S immigrants, the government of Country R can adopt two policies, increasing the penalty charge in the case of detection,  $\varepsilon$ , and increasing the probability of detection in each period of their illegal stay,  $p$ . The results of the comparative static analyses are,

$$d\theta_A/d\varepsilon < 0, d\theta_{GA}/d\varepsilon > 0, \quad (28)$$

$$d\theta_A/dp < 0, d\theta_{GA}/dp > 0. \quad (29)$$

To restrict the inflow of type F immigrants, the government of Country R can also adopt two policies, reducing the probability of success in crossing the border,  $\eta$ , and increasing the necessary cost to obtaining fake visas,  $\mu$ . The results of the comparative static analyses are,

$$d\theta_A/d\mu > 0, d\theta_{GA}/d\mu < 0, \quad (30)$$

$$d\theta_A/d\eta < 0, d\theta_{GA}/d\eta \gtrsim 0. \quad (31)$$

The results above imply that for Country R, there is no definite policy to reduce the total number of illegal immigrants, while internal investigation aimed at type S immigrants will cause positive effects on the average quality of illegal immigrants.

Similarly, in Case B, from the definitions of  $\theta_B$  and  $\theta_{FB}$ , we obtain the following two equations,

$$k(r_0 + r\theta_{FB} - p\varepsilon) - (q_0 - q\theta_{FB}) = 0, \quad (32)$$

$$(kr - q)[\gamma(p) - \eta T]\theta_B = -[\gamma(p) - \eta T](kr_0 - q_0) + \gamma(p)kp\varepsilon - \mu. \quad (33)$$

In Case B, recalling (16-1) and (16-2), we have four equations, (24), (25), (32), and (33), from which four endogenous variables,  $\theta_B, \theta_{FB}, r_0$  and  $q_0$  will be determined.

We note again that from (17-1),  $\gamma(p) < \eta T$  is satisfied in Case B, which implies that the expected duration of a type S immigrant in the host country is shorter than that of a type F immigrant. Thus, a decrease in both  $\theta_B$  and  $\theta_{FB}$  would be sufficient to reduce the total number of illegal immigrants. And increasing  $\theta_{FB}$  with decreasing  $\theta_B$  should be sufficient to enhance the average quality of illegal immigrants staying in Country R at any given point.

To restrict the inflow of type S immigrants, the results of the comparative static analyses are,

$$d\theta_B/d\varepsilon > 0, d\theta_{FB}/d\varepsilon < 0, \quad (32)$$

$$d\theta_B/dp > 0, d\theta_{FB}/dp \geq 0. \quad (33)$$

While to restrict the inflow of type F immigrants, the results of the comparative static analyses are,

$$d\theta_B/d\mu < 0, d\theta_{FB}/d\mu > 0, \quad (34)$$

$$d\theta_B/d\eta > 0, d\theta_{FB}/d\eta < 0. \quad (35)$$

The results above imply that in Case B, there is also no definite policy to reduce the total number of illegal immigrants, while border control to type F immigrants will cause positive effects on the average quality of illegal immigrants.

We now establish the following propositions.

**Proposition 1:**

- 1) Consider Case A, where medium productivity workers migrate illegally as type F

immigrants while lower productivity workers migrate illegally as type S immigrants. Then, internal investigation aimed at type S immigrants will contribute to enhancing the average productivity of illegal immigrants.

2) Consider Case B, where medium productivity workers migrate illegally as type S immigrants while lower productivity workers migrate illegally as type F immigrants. Then, border control policies aimed at type F immigrants will contribute to enhancing the average productivity of illegal immigrants.

3) In either case, there is no definite policy for the host country to reduce the total number of illegal immigrants.

Proposition 1 leads to the following implications. First, in the case where the two types of illegal immigrants co-exist without any legal migration program, it may be impossible to find an effective policy to reduce the total number of illegal immigrants. Second, to enhance the average quality of illegal immigrants, the target of the restriction should be “lower class” illegal immigrants (i.e., type S in Case A and type F in Case B).

#### 4. Legalization of Illegal Workers

Next let us consider the political effects of the introduction of legal immigrants with sufficient levels of productivity, and the legalization of some illegal workers with relatively high productivity. These policies may be considered by the government of Country R to enhance the average quality of foreign workers.

We assume that  $kw_R > w_Q$  for all  $\theta$ , which implies that even the worker with the highest productivity  $\bar{\theta}$  in Country Q prefers to migrate to Country R. Let  $\hat{\theta}$  denote the minimum productivity to be a legal worker in Country R. We consider the case where legal migration and the two types of illegal immigration co-exist, and also consider that legal permission changes the optimal behavior of illegal immigrants with relatively high productivity, that is  $\hat{\theta}$  satisfies the following conditions,  $\theta_A < \hat{\theta} < \theta_{GA}$  in Case A and

$\theta_B < \hat{\theta} < \theta_{FB}$  in Case B.<sup>6</sup>

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<sup>6</sup> If  $\hat{\theta}$  is larger than  $\theta_{GA}$  or  $\theta_{FB}$ , the group of legal migrants and illegal migrants are completely separated and the effects on expanded legal migrants are limited, such as effects caused from a change in wage rates in equilibrium. On the other hand, if  $\hat{\theta}$  is

In Case A, individuals whose productivities falls in the range  $[0, \theta_A]$  prefer to be type S migrants, while those in the range  $[\theta_A, \hat{\theta}]$  prefer to be type F migrants, and those in the range  $[\hat{\theta}, \bar{\theta}]$  prefer to be legal migrants. Recalling (16-1) and (16-2), we have the following three equations,

$$r_0 = r_0(\gamma(p)\frac{r}{2}\theta_A^2 + \eta T\frac{r}{2}(\hat{\theta}^2 - \theta_A^2) + T\frac{r}{2}(\bar{\theta}^2 - \hat{\theta}^2), \quad (36)$$

$$q_0 = q_0(\gamma(p)\frac{q}{2}\theta_A^2 + \eta T\frac{q}{2}(\hat{\theta}^2 - \theta_A^2) + T\frac{q}{2}(\bar{\theta}^2 - \hat{\theta}^2), \quad (37)$$

$$(kr - q)[\gamma(p) - \eta T]\theta_A = -[\gamma(p) - \eta T](kr_0 - q_0) + \gamma(p)kp\varepsilon - \mu, \quad (38)$$

where three endogenous variables,  $\theta_A$ ,  $r_0$  and  $q_0$  are determined in equilibrium.

The results of the expansion of legal workers' in country R are as follows,

$$d\theta_A/d\hat{\theta} > 0, \quad dr_0/d\hat{\theta} > 0, \quad dq_0/d\hat{\theta} < 0, \quad (39)$$

which implies that the expansion of legal immigration (legalization of illegal immigrants with high productivity) by country R will reduce the total number of illegal immigrants, and enhance the average productivity of total immigrants in Case A.

Similarly, in Case B, individuals whose productivities falls in the range  $[0, \theta_B]$  prefer to be type F migrants, while those in the range  $[\theta_B, \hat{\theta}]$  prefer to be type S migrants, and those in the range  $[\hat{\theta}, \bar{\theta}]$  prefer to be legal migrants. Recalling (16-1) and (16-2), we have the following three equations,

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less than  $\theta_A$  or  $\theta_B$ , then only one type of illegal immigrants exists in equilibrium. As these two situations are not interesting, and do not fit the main focus of this study, we only consider the above cases.

$$r_0 = r_0(\eta T \frac{r}{2} \theta_B^2 + \gamma(p) \frac{r}{2} (\hat{\theta}^2 - \theta_B^2) + T \frac{r}{2} (\bar{\theta}^2 - \hat{\theta}^2), \quad (40)$$

$$q_0 = q_0(\eta T \frac{q}{2} \theta_B^2 + \gamma(p) \frac{q}{2} (\hat{\theta}^2 - \theta_B^2) + T \frac{q}{2} (\bar{\theta}^2 - \hat{\theta}^2), \quad (41)$$

$$(kr - q)[\gamma(p) - \eta T] \theta_B = -[\gamma(p) - \eta T](kr_0 - q_0) + \gamma(p)kp\varepsilon - \mu, \quad (42)$$

where three endogenous variables,  $\theta_B$ ,  $r_0$  and  $q_0$  are determined in equilibrium.

The results of the expansion of legal workers in country R are as follows,

$$d\theta_B/d\hat{\theta} > 0, \quad dr_0/d\hat{\theta} > 0, \quad dq_0/d\hat{\theta} < 0, \quad (43)$$

which implies that the expansion of legal immigration (legalization of illegal immigrants with high productivity) by country R will reduce the total number of illegal immigrants, and enhance the average productivity of total immigrants in Case B. This leads us to the following Proposition.

**Proposition 2:**

The expansion of legal immigration (legalization of illegal immigrants with high productivity) in country R will reduce the total number of illegal immigrants and enhance the average productivity of total immigrants in either case.

The above proposition suggests that to control both the total number and average quality of illegal immigrants, introduction and expansion of legal migration with sufficient levels of productivity should be effective for the economy of the host country. The intuitive way to understand this is that decreasing  $\hat{\theta}$  contributes to increasing the number of relatively higher productivity workers because they are now legal immigrants and can stay longer. Decreasing  $\theta_A$  or  $\theta_B$  also causes positive effects on the quality of immigrants because some immigrants' duration in the host country would decrease.

**5. Concluding Remarks**

Our study suggests that restriction policies to illegal immigrants with lower productivity will likely contribute to enhancing the average productivity of immigrant workers. This result is caused by two different effects. The first effect is that some marginal immigrants change their types and stay shorter than before. The second effect is that an increase in the expected income (this direct effect dominates the opposite indirect effect caused by some immigrants staying in the host country for a shorter period as mentioned above) will draw new immigrants with relatively high productivity. On the other hand, restrictions to those with relatively higher productivity by the government of the host country may cause negative effects on the productivity of illegal immigrant workers, even though this policy itself reduces the inflow of targeted illegal immigrants. This result is caused by two different effects. The first effect is that some marginal immigrants change their types and stay longer than before. The second effect is that due to the reduction of expected income, some relatively high productivity workers do not migrate any more.

Additionally, if the government of the host country permits the legal inflow of workers whose productivities satisfy a required level, legalization of illegal immigrants with high productivity will make it possible to reduce the total number of illegal immigrants and enhance the average productivity of total immigrants. However, this outcome assumes that all workers including those with the highest level of productivity in Country Q choose to migrate to Country R regardless of the discount factor of being away from their families. If we assume that these people with the highest level of productivity do not migrate, that is, if we assume that  $kw_R(\bar{\theta}) < w_Q(\bar{\theta})$  holds, Proposition 2 no longer holds. This is because additional legalization of those with relatively high productivity will extend the expected duration of stay of marginal immigrants, which will reduce the wage gap between the two countries, and thus some marginal migrants with the highest productivity will no longer migrate.

For future work, the following research subjects still remain to be explored. First, we assumed that the necessary cost to obtain a fake visa document,  $\mu$ , is independent of the probability of success in crossing the border,  $\eta$ . We can also consider another scenario where  $\eta = \eta(\mu)$  and  $\eta' > 0$ . Second, we considered illegal immigrants to be those with lower productivity, while assuming that those with high productivity tend to avoid illegal migration. This case is valid under the assumption that the wage difference between workers in Country Q is larger than that in Country R. However, we assumed that the only difference between the two countries is capital endowment, and that Country R is a developed country with abundant capital. Therefore, the wage difference

between workers in Country R could be larger than that in Country Q and, contrary to the model in our study, we could consider a case where those with relatively lower productivity avoid migration. Third, in our study we assumed that each legal worker can migrate from the beginning of his life. But it may be possible and better to consider that legal migration needs sufficient skills, similar to Djajic (1989). High productivity individuals can migrate at a younger age but those with lower productivity must acquire the necessary skills in Country Q and can potentially migrate, at a later time only after mastering those skills. In this context, the optimal decision by each potential migrant in Country Q would compare the expected income of legal migration after skill formation with that of uncertain illegal migration from an earlier point in his/her life.

## References

- Bond, E. W. and T-J. Chen, (1987), The Welfare Effects of Illegal Immigration, *Journal of International Economics*, Vol.23, pp.315-328.
- Carter T. J., (1999), Illegal Immigration in an Efficiency Wage Model, *Journal of International Economics*, Vol.49, pp.385-401.
- Djajic, S., (1987), Illegal Aliens, Unemployment and Immigration Policy, *Journal of Development Economics*, Vol.25, pp.235-249.
- Djajic, S., (1989), Skills and the Pattern of Migration: The Role of Qualitative and Quantitative Restrictions on International Labor Mobility, *International Economic Review*, Vol.30, pp.795-809.
- Djajic, S., (1997), Illegal Immigration and Resource Allocation, *International Economic Review*, Vol.28, pp.97-117.
- Ethier, W. J., (1986), Illegal Immigration: The Host Country Problem, *American Economic Review*, Vol.76, pp. 56-71.
- Harris, J. R., and M. Todaro, (1970), Migration, Unemployment and Development: A Two Sector Analysis, *American Economic Review*, Vol.60, pp.126-142.
- Kats, E. and O. Stark, (1984), Migration and Asymmetric Information, *American Economic Review*, Vol.74, pp. 533-534.
- Kondoh, K., (2000), Legal Migration and Illegal Migration: The Effectiveness of Qualitative and Quantitative Restriction Policies, *Journal of International Trade and Economic Development*, Vol.9, pp.227-245.
- MacDougall, G. D. A., (1960), The Benefits and Costs of Private Investment from Abroad: A Theoretical Approach, *Economic Record*, Vol.26, pp.13-35.
- Stark, O., (1991), *The Migration of Labor*, Blackwell.
- Yoshida, C., (1993), The Global Welfare of Illegal Immigration, *Indian Economic Review*, Vol.28, pp.111-115.
- Yoshida, C., (1998), The Effects of Border Patrol on Illegal Immigration, *Seoul Journal of Economics*, Vol.11, pp.61-74.