Chukyo University Institute of Economics

Discussion Paper Series

September 2019

No. 1905

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Abstract

A regulator can observe the total concentration of non-point source (NPS) pollution, however, cannot monitor individual emissions with low cost and high enough accuracy. This information asymmetry makes adequate standard instruments of environmental policy impossible. This paper constructs a simple Cournot competitive model and considers how much the ambient charge tax can control NPS pollution in a three-stage game. It is shown that the sub-game perfect equilibrium is obtained in which the optimal tax is determined to maximize the social welfare at the first stage; the profit maximizing firms adopt the optimal abatement technologies at the second stage and the optimal productions at the third stage. It is also demonstrated that an increase of the ambient tax can decrease the total concentrations not only at the second stage but at the third stage as well.

Keywords: Non-point souce pollution, Ambient charges, Abatement technology, Three-stage game, Cournot competition, Sub-game perfect equilibrium

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*The first and second authors highly acknowledge the financial supports from the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research (C) 18K0163) and from Chuo University (Grant for special research).

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1 Introduction

To date, non-point source (NPS) pollution is widely dispersed in the environment such as farm field surface runoff; water pollution contaminating river, lake and underwater; air pollution that may arise health problems for humans. It is also called diffused pollution or surface pollution and its prominent feature is the diffuse sources with which it is not easy to identity emissions of individual polluters. Consequently, the design of efficient environmental policy is hampered by informational problems associated with the inability to observe individual contribution to the ambient concentration. Due to informational asymmetries, the regulator or the policy-maker cannot use standard environmental policy instruments including emission taxes, tradable permits, subsidies for emission reductions, deposit system, Pigouvian tax, etc. In order to control NPS pollution, Segerson (1988) proposes monitoring ambient concentrations of pollutants. In this approach, the regulator first determines an environmental standard level and then, imposes uniform tax on the pollutants if the concentration is above the standard level and pays uniform subsidies if it is below.

Ganguli and Raju (2012) shows a "perverse" effect of the ambient charges on the total pollution in the Bertrand duopoly, that is, an increase in the ambient charge tax could lead to larger pollution. Further, Raju and Ganguli (2013) consider the ambient charge effect in a Cournot duopoly and numerically show the effectiveness of the ambient charge to control NPS pollution under a two-stage game. Sato (2017) analytically shows that a higher ambient charge reduces pollutant emissions in a Cournot duopoly market. Some n-firm extensions from a duopoly setting have already started. Matsumoto et al. (2018) construct an n-firm Cournot model and reexamine those static results in a dynamic framework in which an equilibrium can lose stability. Ishikawa et al. (2019) turn attention to the ambient charge effect in a n-firm Bertrand model and show that the sign of the effect depends on the number of the firms, the degree of substitutability and the heterogeneity of abatement technology. One important point that the literature has largely left unaddressed is how the regulator determines the ambient charge tax. Although the analysis is limited to a duopoly framework, this paper focuses on this issue in a three-stage Cournot game.

The rest of the paper is organized as follows. Section 2 provides a base model and solves a three-stage game. Accordingly, this section is divided into three subsections. Section 2.1 determines the optimal output levels, given the abatement technologies and the ambient tax rate. Section 2.2 selects the optimal technologies, given the ambient tax rate. Section 2.3 determines the optimal tax rate that maximizes the social welfare. The final section summarizes the results and presents further research directions.

2 Three-stage Game

There are two firms in the duopoly market in which Cournot competition takes place and NPS pollutions are emitted. Firm k produces an amount q_k of homogenous goods for $k = i, j$. The price function is assumed to be linear,

$$p = a - (q_i + q_j). \tag{1}$$

Each firm produces output as well as emits pollutions and it is assumed that one unit of production emits one unit of pollution. However, using an abatement technology ϕ_k, the firm can reduce the actual amount of pollution to $\phi_k q_k$ by abating $(1-\phi_k)q_k$. The technology is subject to $0 \leq \phi_k \leq 1$ with a pollution-free technology if $\phi_k = 0$ (i.e., no pollution) and a fully-discharged technology if $\phi_k = 1$ (i.e., no abatement). The regulator can measure the total emission quantity[1] $\sum_k \phi_k q_k$ but cannot identify individual contributions to the total quantity. To control the ambient concentrations, it enforces the environmental policy that has an exogenously determined environmental standard E and imposes uniform ambient tax rate θ on the polluted emissions, $\sum_k \phi_k q_k$. This θ is measured in some monetary unit per emission and assumed to be positive but is not necessarily less than unity. The regulator will, according to θ times the difference between $\sum_k \phi_k q_k$ and E, levy the penalty if the difference is positive and award the subsidy if negative.[2]

There are three decision variables, the regulator imposes an ambient charge tax with rate θ while firm k makes choices of the abatement technology, ϕ_k and production of output, q_k. In this paper, under the following time structure, three variables are determined one by one in a three-stage Cournot game. At the first stage, the regulator determines the tax rate of ambient charges to maximize the welfare of the people involved in the market. Having known the tax rate, the firms sequentially take actions in two steps. Each firm determines its optimal abatement technology at the second stage. Then it chooses a production level so as to maximize profit at the third stage, using the optimal technology obtained at the second stage and having the ambient tax rate. As usual, solving this three-stage game backwardly, we derive the sub-game perfect equilibrium of the game, that is, we determine first the optimal output levels, given the level of the abatement technologies and the tax rate, then the optimal technologies, given the tax rate and finally the optimal rate that maximizes the social welfare. Before proceeding we make two assumptions only for the sake of analytical simplification.[3]

Assumption 1. (i) $a = 1$; (ii) the firms have zero production costs.

2.1 Third Stage

Under Assumption 1, firm i determines production of output to maximize its profit defined as

$$\pi_i(q_i) = pq_i - \theta(\phi_i q_i + \phi_j q_j - E). \tag{2}$$

[1]"total quantity of pollutions," "total pollutions" and "ambient concentrations" are synonymous in this paper.

[2]Since E is exogeneously given, it can be mentioned that firm i receives subsidy θE and pays taxes of $\theta(\phi_i q_i + \phi_j q_j)$.

[3]In a future study, we will consider the similar game without this assumption.

Substituting the price function (1) into the profit function and differentiating the resultant profit function present the first-order condition for an interior solution,

$$\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - q_j - \theta\phi_i = 0 \tag{3}$$

where the second-order condition is satisfied (i.e., $\partial^2\pi_i/\partial q_i^2 = -2 < 0$). The first-order condition for firm j is similarly obtained. The optimal levels of output can be obtained by solving simultaneously these two first-order conditions, which can be rewritten as

$$\begin{aligned} 2q_i + q_j &= 1 - \theta\phi_i \\ q_i + 2q_j &= 1 - \theta\phi_j. \end{aligned} \tag{4}$$

The optimal production levels of outputs at the third stage are

$$\begin{aligned} q_i^*(\theta, \phi_i, \phi_j) &= \frac{1}{3}\left(1 + \theta\phi_j - 2\theta\phi_i\right), \\ q_j^*(\theta, \phi_i, \phi_j) &= \frac{1}{3}\left(1 + \theta\phi_i - 2\theta\phi_j\right). \end{aligned} \tag{5}$$

For the non-negativity of output, the levels of the abatement technology should satisfy the following inequalities,

$$\phi_i \geq 0,\ \phi_j \geq 0 \text{ and } \frac{1}{2}\phi_i + \frac{1}{2\theta} \geq \phi_j \geq 2\phi_i - \frac{1}{\theta}. \tag{6}$$

We graphically construct the feasible region of ϕ_i and ϕ_j satisfying the conditions in (6) in Figure 1. Zero-production loci of $q_i^* = 0$ and $q_j^* = 0$ are, from (5),

$$\phi_j = 2\phi_i - \frac{1}{\theta} \text{ and } \phi_j = \frac{1}{2}\phi_i + \frac{1}{2\theta}$$

where the former is a straight line with a steeper slope and a horizontal intercept $1/2\theta$ and the latter is also a straight line with a flatter slope and a vertical intercept $1/2\theta$. The two lines cross the diagonal at $(1/\theta, 1/\theta)$. In the region surrounded by these two lines and parts of the horizontal and vertical axises (that is, red-, yellow- and green-colored regions), the conditions (6) are fulfilled and thus $q_i^* \geq 0$ and $q_j^* \geq 0$. (Ignore the dashed curves, two straight lines starting at the origin and the black dot for now).

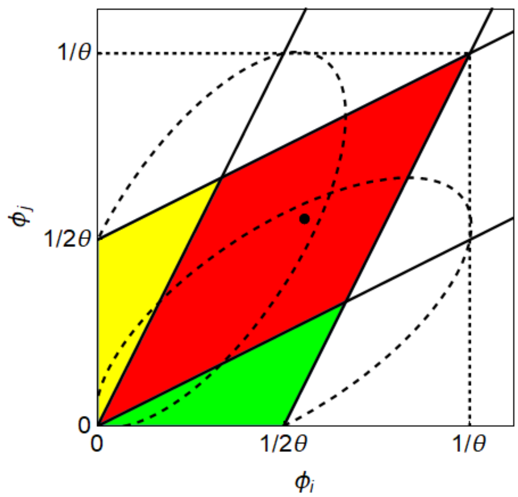

Figure 1. Feasible region of ϕ_i and ϕ_j

The total quantity of pollutions at the Cournot equilibrium is

$$E^*(\theta, \phi_i, \phi_j) = \phi_i q_i^*(\theta, \phi_i, \phi_j) + \phi_j q_j^*(\theta, \phi_i, \phi_j) \tag{7}$$

for which we have the following result[4]:

Theorem 1 *Given non-negative abatement technology, ϕ_k for $k = i, j$, increasing the policy parameter θ decreases the total concentrations,*

$$\frac{\partial E^*(\theta, \phi_i, \phi_j)}{\partial \theta} \leq 0.$$

Proof. *Substituting q_i^* and q_j^* into equation (7) and then differentiating it give*

$$\begin{aligned} \frac{\partial E^*(\theta, \phi_i, \phi_j)}{\partial \theta} &= -\frac{2}{3}\left(\phi_i^2 - \phi_i \phi_j + \phi_j^2\right) \\ &= -\frac{2}{3}\left[\left(\phi_i - \phi_j\right)^2 + \phi_i \phi_j\right] \leq 0 \end{aligned}$$

where the equality holds only for $\phi_i = \phi_j = 0$. ■

Although individually emitted pollutions are non-observable, we are interested in how much each firm responses to the policy change. First, differentiating $\phi_i q_i^*$ for $k = i, j$ with respect to θ gives

$$\frac{\partial}{\partial \theta}\left(\phi_i q_i^*\right) = \frac{1}{3}\phi_i\left(\phi_j - 2\phi_i\right) \text{ and } \frac{\partial}{\partial \theta}\left(\phi_j q_j^*\right) = \frac{1}{3}\phi_j\left(\phi_i - 2\phi_j\right).$$

[4] This is already shown by Sato (2017).

Zero-responses loci of each firm's pollution are described by

$$\phi_j = 2\phi_i \text{ and } \phi_j = \frac{1}{2}\phi_i.$$

These formula are illustrated as straight lines passing through the origin in Figure 1. It is also obtained

$$\frac{\partial}{\partial\theta}\left(\phi_i q_i^*\right) \gtreqless 0 \text{ according to } \phi_j \gtreqless 2\phi_i$$

and

$$\frac{\partial}{\partial\theta}\left(\phi_j q_j^*\right) \gtreqless 0 \text{ according to } \phi_j \lesseqgtr \frac{1}{2}\phi_i.$$

Both responses are negative in the red-colored parallelogram surrounded by four solid lines. Firm i's response is positive and firm j's response is negative in the yellow-colored triangle in the lower-left. Roughly speaking, if an abatement technology of firm i is more efficient than that of firm j, then firm i emits more pollutions and firm j abates more pollutions when the value of θ increases. However decreases in pollutions dominate increases in pollutions, leading to the result that the total concentrations decrease. Responses between the firms are interchanged in the green-colored triangle in the lower-right when firm j with efficient technology emits more pollutions.

We also examine how firm's profit varies as the ambient charge varies. Now ϕ_i and ϕ_j are considered fixed. The maximized profit is obtained by substituting the optimal outputs in (5)

$$\bar{\pi}_i(\theta) = p(\theta)\bar{q}_i(\theta) - \theta(\phi_i\bar{q}_i(\theta) + \phi_j\bar{q}_j(\theta) - E) \tag{8}$$

where the optimal output is simplified as $\bar{q}_i(\theta)$ and $\bar{q}_j(\theta)$ and the corresponding price is denoted by $p(\theta) = 1 - \bar{q}_i(\theta) - \bar{q}_j(\theta)$. Differentiating (8) with respect to θ gives

$$\frac{\partial\bar{\pi}_i(\theta)}{\partial\theta} = \frac{1}{9}\left\{2\theta\left(4\phi_i^2 - 7\phi_i\phi_j + 7\phi_j^2\right) - \left(4\phi_i + \phi_j\right)\right\}. \tag{9}$$

In the same way, the marginal profit of firm j is obtained,

$$\frac{\partial\bar{\pi}_j(\theta)}{\partial\theta} = \frac{1}{9}\left\{2\theta\left(4\phi_j^2 - 7\phi_i\phi_j + 7\phi_i^2\right) - \left(4\phi_j + \phi_i\right)\right\}. \tag{10}$$

The zero-marginal profit of firm i is illustrated as the dotted elliptical shape curve close to the horizontal axis in Figure 1. The marginal profit of firm i is positive outside the curve and negative inside. The zero-marginal profit of firm j is the dotted curve close to the vertical axis. The marginal profit of firm j is also positive outside and negative inside. One of the disadvantages of the ambient charge is a moral hazard problem associated with the asymmetric information. In a situation in which each discharger's emission is not observable, it can increase its profit by choosing a lower level of abatement. When the ambient charge tax increases, firm i increases emissions and profit in the yellow region above the dotted curve in which the abatement technology of firm i is more efficient than that of firm j (i.e., $\phi_i < \phi_j$). On the other hand, firm j increases emissions and profit in the green region right to the dotted curve in which firm j has more efficient abatement technology.

Proposition 1 *When the ambient charge tax varies, moral hazard arises in cases with firms having strongly asymmetric abatement technologies.*

2.2 Second Stage

Given the ambient tax rate θ and the optimal output decisions q_i^* and q_j^* in (5), each firm determines the optimal abatement technology at the second stage. Substituting the optimal outputs into the profit function (2) and subtracting the implementation cost of the abatement technology present the reduced form of the profit function of firm i as

$$\pi_i^*(\phi_i) = (1 - q_i^* - q_j^*)q_i^* - \theta\left(\phi_i q_i^* + \phi_j q_j^* - E\right) - (1 - \phi_i)^2 \tag{11}$$

where θ is considered fixed. The arguments of q_i^* and q_j^* are omitted for notational simplicity. Differentiating (11) with respect to ϕ_i yields the first-order condition,

$$\frac{\partial \pi_i^*}{\partial \phi_i} = \frac{\partial \pi_i^*}{\partial q_i}\frac{\partial q_i^*}{\partial \phi_i} + \frac{\partial \pi_i^*}{\partial q_j}\frac{\partial q_j^*}{\partial \phi_i} + \frac{\partial \pi_i^*}{\partial \phi_i}_{|q_i^*, q_j^* : const} = 0 \tag{12}$$

where

$$\begin{aligned} \frac{\partial \pi_i^*}{\partial q_i} &= 1 - 2q_i^* - q_j^* - \theta\phi_i = 0, \\ \frac{\partial \pi_i^*}{\partial q_j} &= -\ q_i^* - \theta\phi_j, \\ \frac{\partial q_j^*}{\partial \phi_i} &= \frac{\theta}{3}, \\ \frac{\partial \pi_i^*}{\partial \phi_i}_{|q_i^*, q_j^* : const} &= 2(1 - \phi_i) - \theta q_i^*. \end{aligned} \tag{13}$$

The second-order conditions for firms i and j are

$$\frac{\partial^2 \pi_i^*}{\partial \phi_i^2} = \frac{\partial^2 \pi_j^*}{\partial \phi_j^2} = \frac{8}{9}\left(\theta^2 - \frac{9}{4}\right) \leq 0 \tag{14}$$

that is satisfied if

$$0 \leq \theta \leq \frac{3}{2}. \tag{15}$$

Rearranging the terms in (12) simplifies the form of the first order condition for firm i as

$$2(4\theta^2 - 9)\phi_i - 7\theta^2\phi_j = 4\theta - 18. \tag{16}$$

In the same way, the first-order condition for firm j is

$$-7\theta^2\phi_i + 2(4\theta^2 - 9)\phi_j = 4\theta - 18. \tag{17}$$

Solving (16) and (17) simultaneously yields the optimal choice of the abatement technology,

$$\phi_i^* = \phi_j^* = \phi^*(\theta) = \frac{18 - 4\theta}{18 - \theta^2}. \tag{18}$$

Since the firms are symmetric, their optimal choices should be identical. The choice of the optimal technology against θ is illustrated in Figure 2. Notice that the dashed vertical line stands at $\theta_0 = 3\sqrt{2} \simeq 4.243$ and divides the horizontal axis into two parts, $18 - \theta^2 > 0$ for $\theta < \theta_0$ and $18 - \theta^2 < 0$ for $\theta > \theta_0$. To avoid congestion, θ_0 is labelled on the upper horizontal line. Accordingly, as is seen in Figure 2, the optimal abatement technology is determined as

$$\frac{2}{3} \leq \phi^*(\theta) \leq 1 \text{ if } 0 \leq \theta \leq 4 \text{ and } 0 \leq \phi^*(\theta) \leq 1/3 \text{ if } \theta \geq \frac{9}{2}$$

where the first condition for θ is weaker than the SOC in (15) and the second condition violates it. The optimal choices for $\theta \geq 9/2$ is eliminated for further considerations.

Differentiating the optimal technology (18) with respect to θ presents

$$\frac{d\phi^*}{d\theta} = -\frac{4(\theta - 3)(\theta - 6)}{(18 - \theta^2)^2}. \tag{19}$$

The denominator is always positive unless $\theta \neq \theta_0$. On the other hand, the numerator is negative if $3 < \theta < 6$ and non-negative otherwise. Hence the derivative is definitely negative in the shaded region of Figure 2 in which the SOC holds. This is a natural result, implying that the rational firm chooses more effective abatement technology if the regulator imposes heavier ambient charges. Concerning the properties of the optimal abatement technology, we summarize the results obtained so far.

Proposition 2 *The optimal abatement technology is positive and less than unity and decreases as the ambient charges increase, since*

$$\frac{16}{21} < \phi^*(\theta) \leq 1 \textit{ and } \frac{\partial \phi^*}{\partial \theta} < 0 \textit{ for } 0 \leq \theta \leq \frac{3}{2}.$$

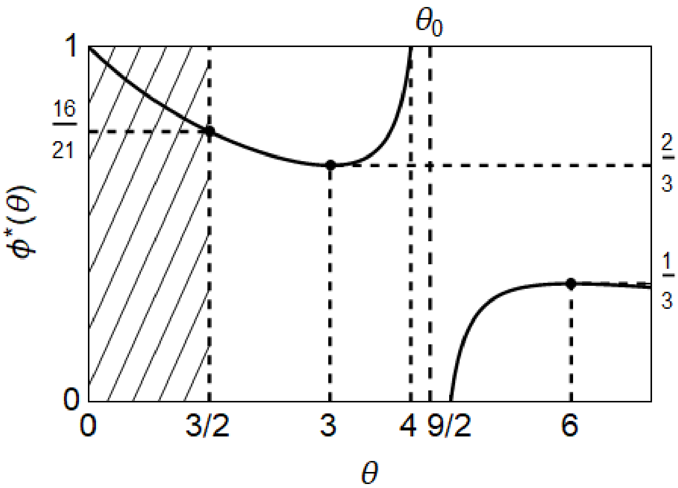

Figure 2. Optinal abatement technology, $\phi^*(\theta)$

Optimal outputs with the optimal technology are obtained by inserting (18) into (5),

$$q_i^*(\theta, \phi_i^*, \phi_j^*) = q_j^*(\theta, \phi_i^*, \phi_j^*) = \frac{\theta^2 - 6\theta + 6}{18 - \theta^2}. \tag{20}$$

Since $\theta \leq 3/2$ is assumed, the denominator is positive. The numerator is non-negative if either $0 \leq \theta \leq 3 - \sqrt{3}$ or $\theta \geq 3 + \sqrt{3}$, where the second case has no economic meaning. The optimal output under the optimal abatement technology denoted as $q^*(\theta)$ has two phases,

$$q^*(\theta) = \frac{\theta^2 - 6\theta + 6}{18 - \theta^2} > 0 \text{ if } 0 \leq \theta \leq 3 - \sqrt{3} \tag{21}$$

or

$$q^*(\theta) = 0 \text{ if } 3 - \sqrt{3} < \theta \leq 3/2. \tag{22}$$

Zero production means that the firms exit the market if a stronger environmental policy is enforced in the sense that $\theta \geq 3 - \sqrt{3} \simeq 1.268$.

Differentiating the optimal output (21) presents

$$\frac{dq^*}{d\theta} = -\frac{6\left[(\theta - 4)^2 + 2\right]}{\left(18 - \theta^2\right)^2} < 0 \text{ for } \theta \geq 0. \tag{23}$$

The negative derivative implies that increasing the tax rate decreases the output, leading to decreased emission of pollution.

Proposition 3 *The optimal output under the optimal abatement technology is positive and negatively sensitive to a change in the tax rate if the environmenat policy is not so strong,*

$$q^*(\theta) > 0 \text{ and } \frac{dq^*}{d\theta} < 0 \text{ for } 0 \leq \theta \leq 3 - \sqrt{3}$$

and the firms exit the market if the policy is strong.

The total amount of pollutions emitted by the two firms at the second stage is the double of individually emitted pollutions since the firms are symmetric,

$$E^{**}(\theta) = 2\phi^*(\theta) q^*(\theta) \tag{24}$$

that is non-negative for $\theta \leq 3 - \sqrt{3}$. We are now concerned with how the total amount changes in response to changes in the environmental policy. To this end, we first differentiate $E^{**}(\theta)$ in (24) with respect to θ to have,

$$\frac{1}{2}\frac{dE^{**}(\theta)}{d\theta} = \frac{d\phi^*}{d\theta}q^* + \phi^*\frac{dq^*}{d\theta} < 0 \tag{25}$$

where the equality is due to (19) and (23). If the regulator decides to increase the ambient charges, then there are two effects on the total concentrations. First, the firms decrease output and get decreased amount of pollutions described by the second term in (25). Second, the firms improve their abatement technology more effective and get decreased amount of pollutions. This is described by the first term. The total effect on the concentration is the sum of these decreases and thus definitely negative. When the ambient charges rise, the total amount of pollutions will fall, implying that the policy is effective for controlling the pollutions. We summarize this as our second main result:

Theorem 2 *Assuming optimal abatement technologies and outputs, the increased tax rate θ decreases the total quantity of NPS pollutions,*

$$\frac{dE^{**}(\theta)}{d\theta} < 0 \text{ for } 0 \leq \theta \leq 3 - \sqrt{3}.$$

Another way to see this result is to rewrite the right hand side of (24) in terms of θ. By inserting (18) and (21) into (24), an alternative form of the total pollutions is obtained,

$$E^{**}(\theta) = \frac{2(18 - 4\theta)(\theta^2 - 6\theta + 6)}{(18 - \theta)^2} \tag{26}$$

which is illustrated in Figure 3.

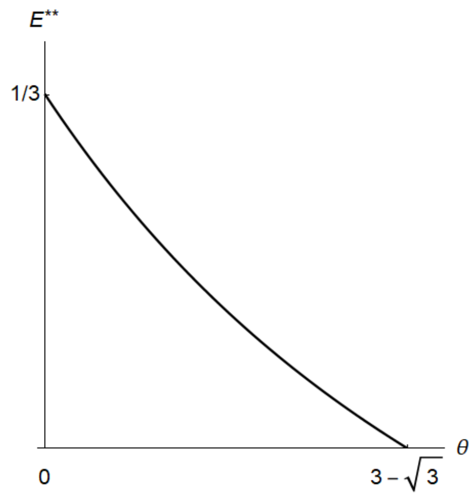

Figure 3. Graph of the total concentrations, $E^{**}(\theta)$

Apparently $E^{**}(\theta)$ has a negative slope and in particular, differentiating (26) with respect to θ gives the following form

$$\frac{dE^{**}(\theta)}{d\theta} = -\frac{4(\theta^4 - 21\theta^3 + 153\theta^2 - 486\theta + 596)}{(18-\theta^2)^3} < 0 \text{ for } 0 \leq \theta < 3 - \sqrt{3}. \quad (27)$$

Yet, in a third way we see Theorem 2 from a different point of view and start with the optimal production (5) with $\phi_i = \phi_j = \phi$,

$$\tilde{q}(\theta, \phi) = \frac{1}{3}(1 - \theta\phi). \quad (28)$$

Partial differentiation yields two derivatives,

$$\frac{\partial \tilde{q}}{\partial \theta} = -\frac{1}{3}\phi < 0 \text{ and } \frac{\partial \tilde{q}}{\partial \phi} = -\frac{1}{3}\theta < 0. \quad (29)$$

The first equation is the *ambient charge effect* that negatively induces changes in outputs. The firms decrease their outputs if a stronger environmental policy is advocated. The second is the *abatement technology effect* that the firms increase outputs if a better abatement technology is set up. These effects are independent as far as $\tilde{q}(\theta, \phi)$ is concerned, however, combined through a change in θ at the optimal point at which the optimal technology depends on θ, $\phi = \phi^*(\theta)$ and the optimal output depends on only θ, $q^*(\theta) = \tilde{q}(\theta, \phi^*(\theta))$. It is differentiated with respect to θ,

$$\frac{dq^*(\theta)}{d\theta} = \frac{\partial \tilde{q}}{\partial \theta} + \frac{\partial \tilde{q}}{\partial \phi}\frac{\partial \phi^*}{\partial \theta}. \quad (30)$$

The first term on the right hand side is the ambient charge effect, itself and the second term describes how much the abatement technology effect is amplified by responsiveness of the abatement technology to a policy change. These terms are substituted into the right hand side of (25) that is reduced to the following form, after arranging the terms:

$$\begin{aligned}\frac{\partial\left[\phi^*(\theta)\,\tilde{q}(\theta,\phi^*(\theta))\right]}{\partial\theta} &= \left[q^*(\theta)+\phi^*(\theta)\frac{\partial\tilde{q}}{\partial\phi}\right]\frac{\partial\phi^*}{\partial\theta}+\phi^*(\theta)\frac{\partial\tilde{q}}{\partial\theta} \\ &= q^*(\theta)\left(1-\varepsilon_q\right)\frac{\partial\phi^*}{\partial\theta}+\left[-\frac{1}{3}\left(\phi^*(\theta)\right)^2\right]\end{aligned} \tag{31}$$

where ε_q denotes the elasticity of output with respect to the abatement technology at the optimal point,

$$\varepsilon_q = -\frac{\phi^*}{q^*}\frac{\partial\tilde{q}}{\partial\phi} = \frac{\theta\phi^*}{1-\theta\phi^*}. \tag{32}$$

Before going to equation (31), we turn a little attention to (32) from which we have

$$\varepsilon_q \gtreqless 1 \text{ according to } \phi^*(\theta) \gtreqless \frac{1}{2\theta}.$$

Solving $2\theta\phi^*(\theta) = 1$ gives the critical value of θ,

$$\theta_1 = \frac{3}{7}\left(6-\sqrt{22}\right) \simeq 0.561.$$

Hence we have

$$\varepsilon_q \gtreqless 1 \text{ according to } \theta \gtreqless \theta_1.$$

The actual quantity of emitted pollutions is the fully-discharged pollutions times the abatement level of the technology. If the ambient charge is altered, then the emitted quantity is affected in two ways, how much pollutions are affected and how much technology is affected. Since it is already assumed that one unit of pollution is equal to one unit of production, to check the change in pollutions is equivalent to check the change in output. The change in output is given in (26) that is the sum of changes caused by the ambient effect and the technology effect. However, as seen in the right hand side of the first line in (31), the total effect is rearranged and is divided into the second terms (i.e., the pollution effect) and the first term (i.e., the extended technology effect). The second term describes how much of changes in pollutions (i.e., output) is remained after abating. The first term is roughly equal to the improvement of technology times changes in pollution induced by changing the technology. The form in the second line of (32) simplifies these effects. If the regulator changes the policy, then the pollution effect is negative whereas the extended technology effect is negative if pollution is inelastic (i.e., $\varepsilon_q < 1$) and positive if elastic (i.e., $\varepsilon_q > 1$). However we have already confirmed in Theorem 2 that even if pollution is elastic, the pollution effect dominates the technology effect.

In Figure 4 (ignore the θ^e for now), the extended technology effect is illustrated by the upward-sloping solid curve, the pollution effect by the downward-sloping solid curve and the sum of these effects by the upward-sloping dotted curve. It can be seen first that the upward-sloping curve crosses the horizontal axis at $\theta = \theta_1$ where $\varepsilon = 1$[5] and thus the technology effect is positive for $\theta_1 < \theta < \theta_0$. Second, the dotted curve is located below the horizontal axis as the pollution effect dominates the technology effect.

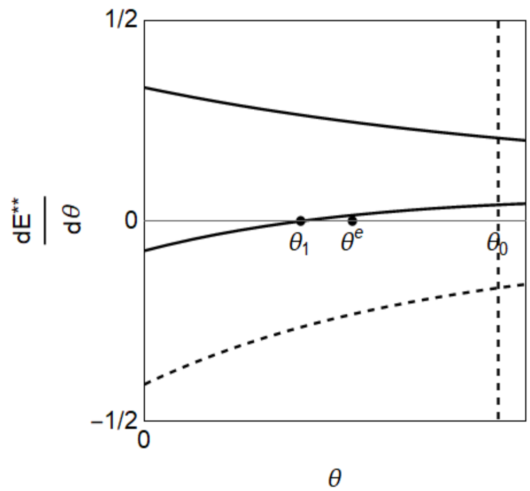

Figure 4. Effects on the total concentrations

2.3 First Stage

The regulator determines the optimal rate of ambient charges to maximize the social welfare function that is defined as

$$W = CS + PS + T - D \tag{33}$$

where CS, PS, T and D stand for consumer surplus, producers surplus (i.e., profit), tax collected associated with pollution emission and the damage caused by NPS pollutions, respectively. Each of which is defined as follows,

$$\begin{aligned} CS(\theta) &= \int_0^{Q^*(\theta)} (1-Q)\, dQ - P^*(\theta)\, Q^*(\theta), \\ PS(\theta) &= P^*(\theta)\, Q^*(\theta) - 2\theta E^*(\theta) - 2(1-\phi^*)^2, \\ T(\theta) &= 2\theta E^*(\theta), \\ D(\theta) &= E^*(\theta) \end{aligned} \tag{34}$$

[5]It also crosses at $\theta = 3$ where $\varepsilon_q > 1$.

where

$$Q^*(\theta) = 2q^*(\theta). \tag{35}$$

Notice that the damage function has the simplest form to simplify the analysis. Hence the form of the welfare function is reduced to

$$W(\theta) = 2q^*(\theta) - 2[q^*(\theta)]^2 - 2(1-\phi^*(\theta))^2 - 2\phi^*(\theta)q^*(\theta). \tag{36}$$

Differentiating (25) with respect to θ yields

$$\frac{dW}{d\theta} = 2(1 - 2q^*(\theta) - \phi^*(\theta))\frac{dq^*(\theta)}{d\theta} + 2(2(1-\phi^*(\theta)) - q^*(\theta))\frac{d\phi^*(\theta)}{d\theta}.$$

Using (18), (19), (21) and (23), this derivative can be reduced to

$$\frac{dW}{d\theta} = \frac{4[(5\theta-24)(3\theta^3 - 26\theta^2 + 66\theta - 36)]}{(18-\theta^2)^3}. \tag{37}$$

Solving the first order-condition for the welfare maximization, $dW/d\theta = 0$, yields two real solutions, $\theta_1 \simeq 0.746$ and $\theta_2 = 24/5$ which satisfy $\theta_1 < 3-\sqrt{3} < \theta_2$. Hence the result is summarized as follows.

Theorem 3 *The optimal rate of ambient charges is determined as*

$$\theta^e \simeq 0.746 < \theta_0 (= 3 - \sqrt{3})$$

and in consequence, the optimal abatement technology and the optimal output are

$$\phi^e = \phi^*(\theta^e) \simeq 0.861 \text{ and } q^e = q^*(\theta^e) \simeq 0.119.$$

All solutions are confirmed to be feasible. Figure 1 is illustrated with $\theta = \theta^e$ and the black dot there corresponds to the point (θ^e, θ^e) that is in the red region, implying that $q^*(\theta^e, \phi^e, \phi^e)$ is positive. In Figure 2, it can be checked that $\phi^e > 16/21 \simeq 0.762$. In Figure 4, it is seen that $\theta^e > \theta_1$ implying $\varepsilon_q > 1$.

3 Concluding Remarks

In this paper, we consider whether the ambient charge controls NPS pollution. Solving a three-stage game, we determine the optimal level of the ambient charge, the optimal levels of abatement technology and the optimal levels of output. We also show that the ambient charge is effective for controlling NPS pollution.

An ambient tax could be effective to control NPS pollution. However, it has some disadvantages with monitoring. Ambient concentrations should be monitored at an acceptable level of accuracy from a fairness point of view and at a lower cost from a practical point of view. Conversely, monitoring is highly expensive and is very difficult to improve the accuracy level. Further, ambient concentrations have natural violability associated with weather condition and abatement technological uncertainty. It uniformly charges all pollutants, some of who are acting in good faith to reduce pollution levels and some others who do not operate the abatement technology at a non-desired level leading to more emissions. Apparently this could lead to a unfairness problem. These are future tasks to be urgently done.

Concerning the technical aspect of the model analysis, we impose strong assumptions to simplify the analysis. Needless to say, relaxing some of them will be done in a next paper. It is also an interesting task to generalize this duopoly model to n-firm Cournot and Bertrand oligopolies.

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