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Abstract
As one countermeasure against the COVID-19 pandemic, governments in most countries are now engaging in large-scale fiscal transfers, which are mostly financed by debt. With huge amounts of government debt already accumulated, governments might partially default on its debt by debt repudiation. We study the long-term effects of debt repudiation on fiscal sustainability using an endogenous growth setting. Debt repudiation lowers the sustainable debt–capital ratio if the initial repudiation ratio is sufficiently low. It also enhances fiscal sustainability by increasing the unstable stationary debt–capital ratio. However, if the initial repudiation ratio is not low, then the increased debt repudiation increases the debt-capital ratio, raising the income tax rate, and degrades fiscal sustainability.

Keywords: debt repudiation, fiscal sustainability, partial default

JEL Classification: E62, G28, H63

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1 Introduction

Facing temporary income declines and increasing temporary government expenditure, governments have tended to rely more heavily on bond issuance in the 1970s and after. Since early 2020, as one countermeasure against the COVID-19 pandemic, governments in most countries have been engaging in large-scale fiscal transfers to support economic activities during and after lockdowns.[1] Such expenditure have been mostly financed by debt. Many economically developed countries have already accumulated huge amounts of sovereign debt, especially after the world financial crisis in 2007–2008. Government gross debt as a percentage of GDP is projected to increase from 105.2% in 2019 to 125.6% in 2021 for advanced economies: specifically from 108.7% to 133.6% in the US, 84.1% to 100.0% in the Euro Area, and 238.0% to 264.0% in Japan (IMF, 2020b).[2] Governments are expected to improve fiscal balances later through normal fiscal measures.

Nevertheless, facing enormous amounts of government debt, governments might partially default on its debt, for instance, through fiscal inflation and/or interest income taxation, i.e., by debt repudiation.[3] If a central bank does not monetize the deficits, then risen interest rates on government debt could increase the probability of default. Consequently, the central bank will have little choice but to monetize them.[4] Monetization can solve several problems for a government during the COVID-19 crisis by reducing to some extent the value of its outstanding obligations. Calvo (1988) uses a two-period model to demonstrate that, with moderate interest rates, it is optimal to resort partially to higher inflation or currency devaluation, i.e., debt repudiation. Arellano et al. (2020) also report that debt relief is useful for avoiding deep debt crises

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[1] The physical and financial effects of COVID-19 on the economy have been analyzed by many researchers (e.g., Atkeson, 2020; Eichenbaum et al., 2020; Guerriri et al., 2020; Faria-e-Castro, 2020; Arellano et al., 2020). These studies are mostly concerned with short-term tradeoff of optimal lockdown policies between saving lives and preventing economic recession until a vaccine becomes available (i.e., in 1.5 or 2 years).

[2] Fiscal support was also higher in Japan than in Western European countries as of June 12 (IMF, 2020a).

[3] The Bank of Japan has financed the debt purchases through reserve creation for the past decades. Oguro (2016) asserts the possibility that such a policy as savings blockade might be undertaken by the Japanese government in the near future. Bloomberg (2020) describes that a top government official said that the European Central Bank should consider wiping out or holding forever the government debt it buys during the current crisis to help nations recover and restructure. Sato (2020) and Landais et al. (2020) suggest a one-time or time-limited wealth tax to repay for the COVID debt. Landais et al. (2020) describe that a wealth tax is preferable to inflation and the less likely to harm growth.

[4] The percent of central government marketable securities or debt issued since February 2020, which are purchased by the central bank, is respectively 57% in the US, 71% in the Euro Area, and 75% in Japan (IMF, 2020b).

and for saving more lives from the COVID-19 in a small open economy.

Debt repudiation has been analyzed by many authors since Eaton and Gersovitz (1981). In the light of the European sovereign debt crisis, the government solvency became an active economic concern again recently. Most arguments are concerned with the interaction between sovereign governments and international financial markets.[5] Few analyses have been conducted in endogenous economic growth settings.[6] We present analyses of long-term effects of debt repudiation policy on the debt–GDP ratio, economic growth, and fiscal sustainability in an endogenous growth model à la Bräuninger (2005). The main finding is that debt repudiation might degrade fiscal sustainability, might increase the income tax rate, and might deter economic growth unless the initial debt repudiation ratio is sufficiently low.

The next section introduces a model. Section 3 presents analyses of debt repudiation effects on fiscal sustainability. Section 4 presents a numerical example. The final section concludes the paper.

2 Model

We assume an overlapping-generations model populated by identical two-period-lived agents. Each agent works and consumes some wage income, saving the remainder during the young period, and when older, retires to consume the fruits of personal savings. The lifetime budget constraint of an agent working in period t is written as

$$(1-\tau_t)w_t = c_t^1 + c_{t+1}^2 / [1+(1-\tau_{t+1})r_{t+1}], \tag{1}$$

where c_t^1 and c_{t+1}^2 respectively represent consumption during the young and retired period; $\tau_t \in (0,1)$ denotes the income tax rate in period t; also w_t and r_{t+1} respectively stand for the wage rate in period t and the interest rate in period $t+1$. Each agent supplies a unit of labor during the young period. Individuals choose consumption to maximize lifetime utility $u_t = \gamma \ln c_t^1 + \delta \ln c_{t+1}^2$ ($\gamma, \delta > 0$ and $\gamma + \delta = 1$). From the first-order conditions we have

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[5] However, Kirsch and Rühmkorf (2017) and Arellano and Bai (2017), for instance, assume away capital accumulation.

[6] In endogenous growth settings with productive government spending, Futagami et al. (2008) analyze effects of a debt–capital target and Yakita (2008) analyzes fiscal sustainability for a constant debt–GDP ratio. However, they did not consider the effects of default.

$$c_t^1 = \gamma(1-\tau_t)w_t \text{ and } s_t(\equiv (1-\tau_t)w_t - c_t^1) = \delta(1-\tau_t)w_t, \tag{2}$$

where s_t represents the lifecycle savings.

Aggregate production technology is assumed to be a Cobb–Douglas function $Y_t = AK_t^{\alpha}(E_t N_t)^{1-\alpha}$ ($A > 0$ and $\alpha \in (0,1)$), where Y_t, K_t and N_t respectively represent the aggregate output, capital and labor in period t. Labor efficiency E_t is assumed to be proportional to the amount of capital per worker, i.e., $E_t = \tilde{\eta}(K_t / N_t)$ ($\tilde{\eta} > 0$). The aggregate production function reduces to $Y_t = A\eta K_t$, where $\eta = \tilde{\eta}^{1-\alpha}$. Parameter $\tilde{\eta}$ measures labor efficiency, reflecting the (average) worker's health condition. [7] Profit-maximization in competitive markets engenders the following conditions:

$$r_t = \alpha\eta A \text{ and } w_t = (1-\alpha)(Y_t / N_t). \tag{3}$$

The factor price equals marginal productivity.

For simplicity we assume that government expends a constant proportion of GDP on goods gY_t, where $g \in (0,1)$. Government borrows bY_t in each period, where b is constant. We also assume plausibly that $g > b$. Letting D_t be the public debt in period t, then the dynamics of public debt becomes

$$D_{t+1} = (1-\theta)D_t + bY_t, \tag{4}$$

where θ denotes the debt repudiation ratio ($\theta \in [0,1]$).[8] For analyses, the repudiation ratio is assumed to be kept constant over time. The interest payment on public debt becomes $r_{bt}(1-\theta)D_t$, where r_{bt} stands for the interest rate which debt-holders receive (hereafter called the debt interest rate). The government levies a flat-rate income tax on factor income and debt income. Therefore, the government budget constraint is given as

$$D_{t+1} - D_t + T_t = gY_t + (1-\theta)r_{bt}D_t + \beta\theta r_{bt}D_t, \tag{5}$$

where $T_t = \tau_t[Y_t + (1-\theta)r_{bt}D_t]$. Tax rate τ_t is determined endogenously to satisfy the budget constraint (5). Parameter β stands for the per-capita cost per unit of repudiated debt ($\beta \in [0,1)$). Following Calvo (1988), the cost can be regarded as transactions costs

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[7] Fornaro and Wolf (2020) assume that COVID-19 lowers labor efficiency growth. Although the effect is not necessarily permanent, it is persistent. Moreover, they assume that it is permanent. Lockdowns might cause long-term human capital accumulation losses. Decreases in labor efficiency are shown to increase the debt–capital ratio and degrade fiscal sustainability.

[8] Following Calvo (1988), repudiation includes anything from open repudiation to a tax on interest. He also describes that inflation and the degree of repudiation are indistinguishable.

associated with debt repudiation.[9]

Finally, the capital market balance is given as $S_t = K_{t+1} + D_{t+1}$, where $S_t = \delta(1-\tau)w_t N_t = \delta(1-\tau)(1-\alpha)Y_t$ is the aggregate savings. The condition can be rewritten as

$$\delta(1-\tau_t)(1-\alpha)Y_t = K_{t+1} + D_{t+1}. \qquad (6)$$

Because physical capital and public debt are perfect substitutes for investors, the following arbitrage condition holds as

$$(1-\theta)r_{bt} = r_t. \qquad (7)$$

This condition determines the debt interest rate.

Now we have the dynamic system. Equation (4) and the government budget constraint (5) give the tax rate as

$$1-\tau_t = [1+b-g-\theta(\frac{1}{\eta A}+\frac{\alpha\beta}{1-\theta})x_t]/(1+\alpha x_t), \qquad (8)$$

where $x_t \equiv D_t / K_t$. Inserting (8) into (6), and rearranging terms, we obtain

$$K_{t+1}/K_t = [\frac{1+b-g-\theta(\frac{1}{\eta A}+\frac{\alpha\beta}{1-\theta})x_t}{1+\alpha x_t}(1-\alpha)\delta - b]\eta A - (1-\theta)x_t. \qquad (9)$$

From (4) and using the production function, we have

$$D_{t+1}/D_t = (1-\theta) + b\eta A / x_t. \qquad (10)$$

Because $(D_{t+1}/D_t)/(K_{t+1}/K_t) = x_{t+1}/x_t$, we can obtain the steady-state debt–capital ratio $x = x_t = x_{t+1}$ from (9) and (10), satisfying

$$(1-\theta) + \frac{b\eta A}{x} = [\frac{1+b-g-\theta(\frac{1}{\eta A}+\frac{\alpha\beta}{1-\theta})x}{1+\alpha x}(1-\alpha)\delta - b]\eta A - (1-\theta)x. \qquad (11)$$

The characteristics of the dynamic system are fundamentally identical to those described by Bräuninger (2005). For analytical purposes, we assume a case of two steady states: one with lower debt–capital ratio x^s, which is stable; and another with high debt–capital ratio x^u, which is unstable (see Appendix A1). In the sense that the debt–capital ratio goes to infinity if the initial debt–capital ratio is greater than x^u, the unstable ratio is crucially important for fiscal sustainability.

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[9] When repudiation is open, it includes legal fees, etc. Alesina et al. (1992) list reputation losses, income redistribution and financial instability as default costs.

3 Long-Term Effect of Debt Repudiation

This section presents analyses of the effects of a debt repudiation policy on fiscal sustainability by considering the stationary debt–capital ratio. For analytical purposes, we assume that the policy change is maintained forever.[10] Defining

$$p(x;\theta) = (1-\theta)(1+x) + \frac{b\eta A}{x} \quad \text{and} \tag{12}$$

$$q(x;\theta,\beta) = [\frac{1+b-g-\theta(\frac{1}{\eta A}+\frac{\alpha\beta}{1-\theta})x}{1+\alpha x}(1-\alpha)\delta - b]\eta A, \tag{13}$$

we have $p(x;\theta) = q(x;\theta,\beta)$ in the steady state (equation (11)). Therefore, we obtain the effects of changes in the repudiation ratio on the debt–capital ratio as $dx/d\theta = (\partial p/\partial\theta - \partial q/\partial\theta)/(\partial q/\partial x - \partial p/\partial x)$. From the stability condition, we can readily demonstrate that $\partial q/\partial x - \partial p/\partial x > 0$ in the stable steady state and $\partial q/\partial x - \partial p/\partial x < 0$ in the unstable steady state. From (12) and (13) we have

$$\frac{\partial p}{\partial\theta} - \frac{\partial q}{\partial\theta} = -(1+x) + \frac{(1-\alpha)\delta x}{1+\alpha x}[1+\frac{\alpha\beta\eta A}{(1-\theta)^2}]. \tag{14}$$

The sign of (14) is ambiguous in general. We consider two cases: (i) the initial repudiation ratio θ is sufficiently small or close to zero, and (ii) the initial repudiation ratio θ is sufficiently great and close to one.[11]

In case (i), for plausible parameters, we can show that the right-hand side of (14) is negative. Therefore, we have $dx^s/d\theta < 0$ and $dx^u/d\theta > 0$. The increased repudiation ratio decreases the stable steady-state debt–capital ratio. It enhances fiscal sustainability in the sense that it increases the crucially important debt–capital ratio. In case (ii), the right-hand side of (14) becomes positive for any $x > 0$. Therefore, we have $dx^s/d\theta > 0$ and $dx^u/d\theta < 0$. An increase in the repudiation ratio increases the debt–capital ratio. It degrades fiscal sustainability. These cases are presented in Figure 1. The following proposition is obtained.

Proposition 1 *Effect of an increase in the debt repudiation ratio on the debt–capital ratio depends on the initial repudiation ratio.*

(*a*) *If the initial repudiation ratio is sufficiently small (great), then the increased*

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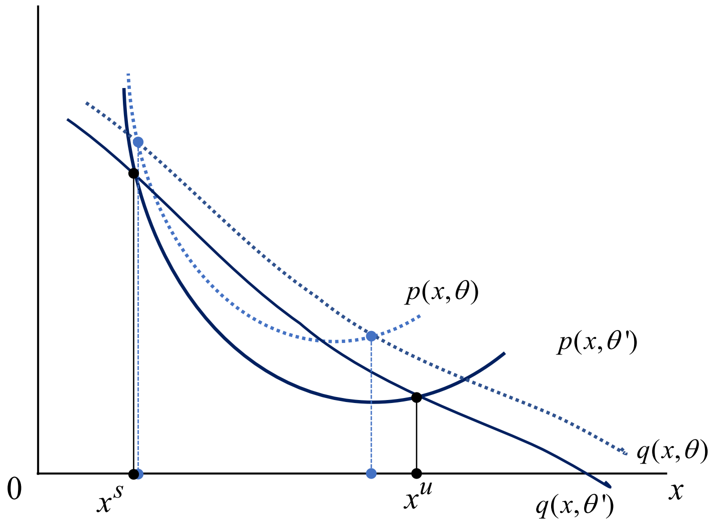
[10] More realistic possibility is that the policy is one-time or time-limited debt repudiation.

[11] Effects also depend on β, although we do not report that here. Section 4 discusses the effects of the repudiation cost in a numerical example.

repudiation engenders a lower (higher) steady-state debt–capital ratio.
(b) *At low (high) initial repudiation ratios, debt repudiation enhances (harms) the fiscal sustainability.*

Proof: See Appendix A2.

(i) Low initial repudiation ratio



(ii) High initial repudiation ratio

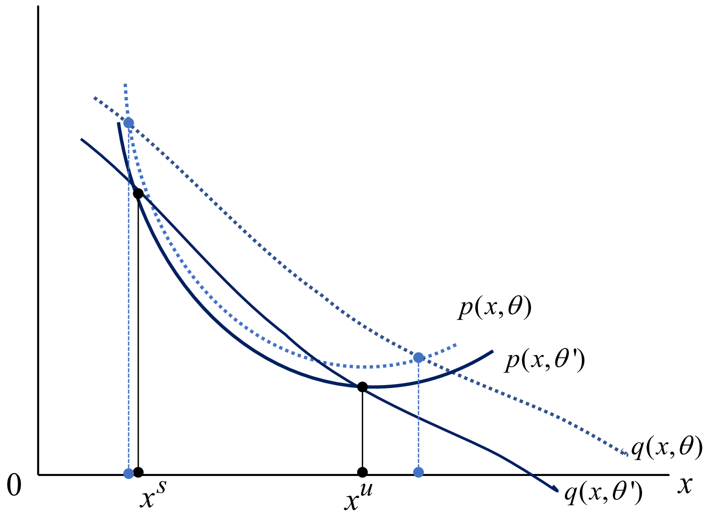


Figure 1. Effects of debt repudiation.

The intuition underlying this result is simple. When the initial debt repudiation ratio is small, the increased debt repudiation reduces the income tax rate largely because of the lower repudiation costs (see (5)). When the repudiation ratio is low, the debt interest rate is also low (see (7)). Consequently, capital accumulation is stimulated. However, if the initial repudiation ratio is high, then the increased repudiation ratio tightens the budget constraint because the repudiation costs are great. As a result, the increased income tax rate decelerates capital accumulation, raising the debt–capital ratio.

In a steady state, from (8), we have

$$d\tau / d\theta = \partial\tau / \partial\theta + (\partial\tau / \partial x)(dx / d\theta), \tag{15}$$

where

$$\frac{\partial\tau}{\partial\theta} = \frac{x}{1+\alpha x}[\frac{1}{\eta A} + \frac{\alpha\beta}{(1-\theta)^2}] > 0 \text{ and} \tag{16a}$$

$$\frac{\partial\tau}{\partial x} = \frac{1}{(1+\alpha x)^2}[\theta(\frac{1}{\eta A} + \frac{\alpha\beta}{1-\theta}) + \alpha(1+b-g)] > 0. \tag{16b}$$

An increase in the repudiation ratio increases the long-term tax rate.[12] The increased debt–capital ratio increases the tax rate. Therefore, when $dx / d\theta > 0$, one obtains $d\tau / d\theta > 0$. By contrast, when $dx / d\theta < 0$, the effect of debt repudiation on the tax rate is ambiguous. When the negative effect of the debt–capital ratio is sufficiently great, the tax rate becomes lower, i.e., $d\tau / d\theta < 0$. The tax rate effect depends on the relative magnitudes of the positive direct effect and the negative indirect effect. We therefore have the following proposition.

Proposition 2 *Whether an increase in the debt repudiation ratio increases the long-term income tax rate, or not, depends on the direct effect and the indirect effect through changes in the debt–capital ratio. When the initial repudiation ratio is sufficiently low, its increase might lower the income tax rate.*

If an increase in the debt repudiation ratio sufficiently lowers the debt–capital ratio when the initial repudiation ratio is sufficiently low, then the increased debt repudiation lowers the income tax rate. If the increased repudiation ratio insufficiently lowers the debt–capital ratio, or if it increases the debt–capital ratio, then the tax rate becomes higher to balance the government budget.

The balanced growth rate γ is given by (9), i.e., $\gamma = K_{t+1} / K_t = Y_{t+1} / Y_t$. From (9) we have

$$d\gamma / d\theta = \partial\gamma / \partial\theta + (\partial\gamma / \partial x)(dx / d\theta), \tag{17}$$

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[12] Policy effects can be considered only for the stable debt–capital ratio.

where

$$\frac{\partial\gamma}{\partial\theta}=\{-\frac{(1-\alpha)\delta\eta A}{1+\alpha x}[\frac{1}{\eta A}+\frac{\alpha\beta}{(1-\theta)^2}]+1\}x \quad \text{and} \tag{18a}$$

$$\frac{\partial\gamma}{\partial x}=-\frac{(1-\alpha)\delta\eta A}{(1+\alpha x)^2}[\theta(\frac{1}{\eta A}+\frac{\alpha\beta}{1-\theta})+\alpha(1+b-g)]-(1-\theta)<0\,. \tag{18b}$$

The sign of the direct effect $\partial\gamma/\partial\theta$ is ambiguous *a priori*. An increase in the debt repudiation ratio does not necessarily raise the balanced growth rate. When the initial repudiation ratio is sufficiently low, the sign of the direct effect might be positive. If the initial repudiation ratio is sufficiently high, then the sign of the direct effect is negative. The indirect effect, which is represented by the second term of (17), is negative (positive) when the initial repudiation ratio is sufficiently low (high). From the results of Proposition 2, when the initial repudiation ratio is sufficiently low, then increases in the repudiation ratio might lower the tax rate, and thereby capital accumulation might be accelerated (i.e., the positive direct effect). If this capital accumulation effect is sufficiently great to overwhelm the negative indirect effect, then the increased repudiation ratio raises the balanced growth rate. In contrast, if the initial repudiation ratio is sufficiently great and close to unity, then increases in the repudiation ratio affect the balanced growth rate negatively.

Proposition 3 *When the initial debt repudiation ratio is sufficiently high and close to unity, then increases in the debt repudiation ratio lowers the balanced growth rate. In contrast, if the initial repudiation ratio is sufficiently low, then the increased repudiation ratio might boost balanced economic growth.*

4 Numerical Example

We consider a numerical example for expositional purposes. Following Bräuninger's (2005) numerical example, we set model parameters as $(\alpha,\delta,g,A,b)=(0.2,0.4,0.2,12,0.02)$. We assume that $\eta=0.95$. Regarding $(\theta,\beta)=(0,0)$ as a benchmark case, we consider cases of $(\theta,\beta)=(0.05,0.1)$ and of $(\theta,\beta)=(0.05,0.8)$. These three cases are presented in Figure 2. Curves $p(x;\theta)$ and $q(x;\theta,\beta)$ shift downward when the repudiation ratio increases. Consequently, an increase in the repudiation ratio θ from 0 to 0.05 decreases the stable debt–capital ratio and increases the unstable debt–capital ratio. The shift of curve $q(x;\theta,\beta)$ is greater for the case of $\beta=0.8$ than for the case of $\beta=0.1$. The repudiation cost per debt also affects the long-term equilibria. Stationary states (x^s,x^u) changes from

(0.1493,1.03) to (0.1433,1.10) when $\beta = 0.1$ and to (0.1437,1.08) when $\beta = 0.8$. The repudiation cost per debt tends to increase the stable debt–capital ratio and decreases the unstable one. It also degrades fiscal sustainability although its effect is not too great to overwhelm the repudiation effect.

The effects of the increase in the debt repudiation ratio on the tax rate and the balanced growth rate are summarized in Table 1. When the repudiation ratio increases, the tax rate becomes lower if $\beta = 0.8$ and higher if $\beta = 0.1$ than in the benchmark case. However, it is noteworthy that debt repudiation raises the balanced growth rate when the initial repudiation ratio is zero. The growth rate becomes higher when $\beta = 0.1$ than when $\beta = 0.8$.

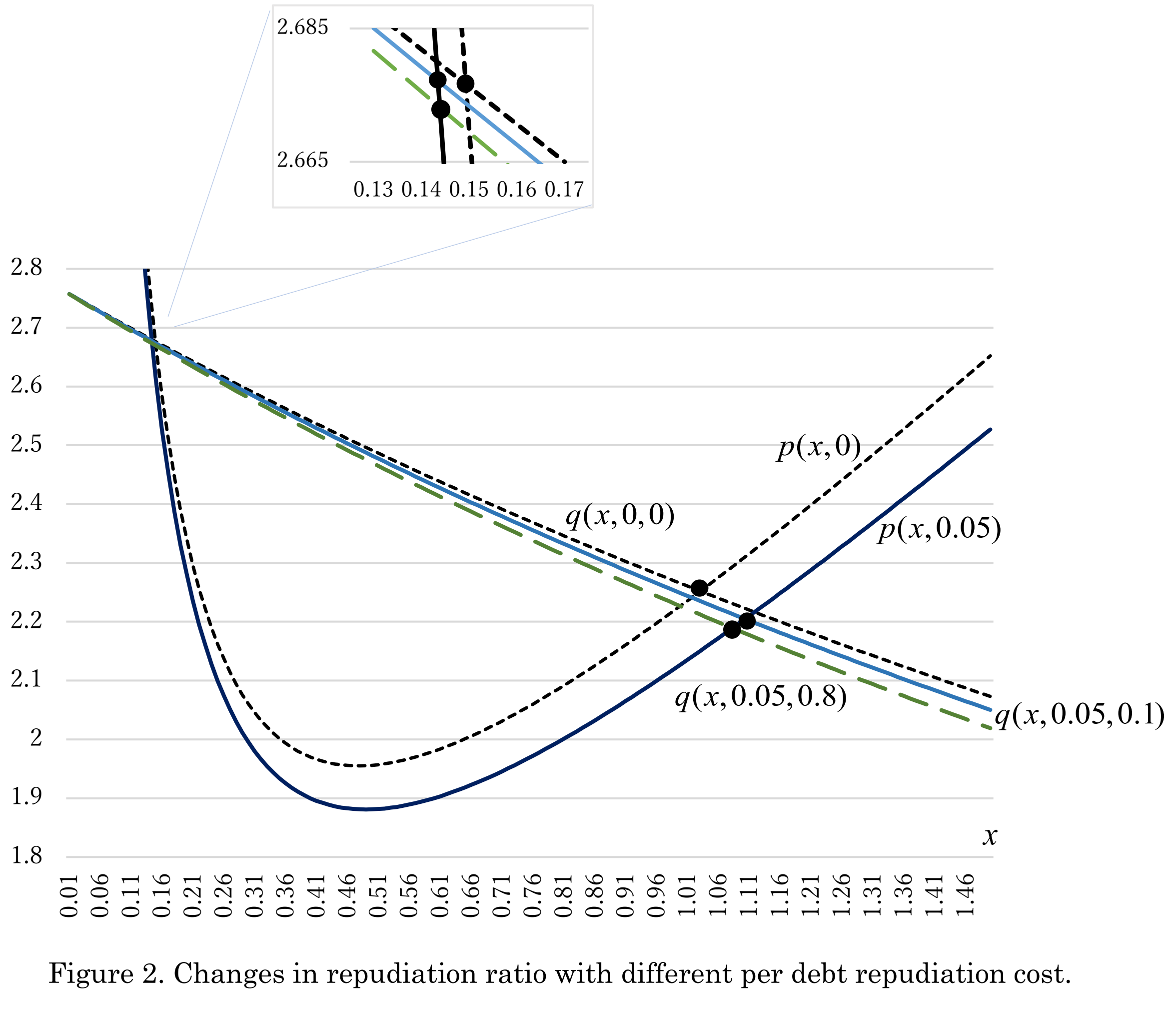

Figure 2. Changes in repudiation ratio with different per debt repudiation cost.

Table 1. Effects on the tax rate and the balanced growth rate

	debt–capital ratio x^S	tax rate τ	annual growth rate γ_{annual} *
$\theta = 0$ (benchmark)	0.1493	0.20378	0.03436
$\theta = 0.05$; $\beta = 0.1$	0.1433	0.20470	0.03448
$\theta = 0.05$; $\beta = 0.8$	0.1437	0.20360	0.03444

* 1period=30years.

5 Concluding Remarks

Assuming that fiscal support for economic activities damaged by the COVID-19 pandemic might have long-term effects on the fiscal balance, we have considered the possible long-term effects of government debt repudiation on economic growth and fiscal sustainability. In the endogenous growth framework, it can be shown that if the initial repudiation ratio is sufficiently small, then additional debt repudiation lowers the long-term debt–capital ratio and might lower the income tax rate in the long term, thereby enhancing fiscal sustainability. However, if the initial repudiation ratio is higher, then the increased debt repudiation increases the long-term dept–capital ratio, degrading fiscal sustainability. Increases in the debt repudiation ratio might not increase the balanced growth rate even when the initial ratio is sufficiently low.

Relaxing the simplifying assumptions and reflecting real factors in the model are subjects of future research. Government expenditure might increase the productivity in the private sector. Explicit consideration of financial sectors enables us to understand more realistic mechanisms for debt repudiation. External sectors should be explicitly considered even if government debt is mostly held domestically.[13]

Appendices

A1 Stability of the Steady State

We examine the stability of the steady state. From (9) and (10) we obtain

$$\frac{x_{t+1}}{x_t} = \frac{(1-\theta) + \dfrac{b\eta A}{x_t}}{[\dfrac{1+b-g-\theta(\dfrac{1}{\eta A}+\dfrac{\alpha\beta}{1-\theta})x_t}{1+\alpha x_t}(1-\alpha)\delta - b]\eta A - (1-\theta)x_t} \equiv \frac{\Delta_S}{\mathrm{K}_S}. \quad \text{(A1)}$$

[13] More than 90 percent of government debt is held domestically in Japan in 2020 (Ministry of Finance, Japan).

By differentiating (A1), we obtain

$$\frac{dx_{t+1}}{dx_t} = 1 + \frac{x}{\mathrm{K}_S}\left(\frac{d\Delta_S}{dx} - \frac{d\mathrm{K}_S}{dx}\right), \tag{A2}$$

where

$$\frac{d\Delta_S}{dx} = -\frac{b\eta A}{x^2}, \tag{A3a}$$

$$\frac{d\mathrm{K}_S}{dx} = -(1-\theta) - \frac{(1+b-g)\alpha + (\frac{1}{\eta A} + \frac{\alpha\beta}{1-\theta})\theta}{(1+\alpha x)^2}(1-\alpha)\delta\eta A, \tag{A3b}$$

and $\Delta_S = \mathrm{K}_S$ in a steady state. Therefore, it follows that

$$\left(\frac{d\Delta_S}{dx} - \frac{d\mathrm{K}_S}{dx}\right)x = [1-\theta + \frac{(1+b-g)\alpha + (\frac{1}{\eta A} + \frac{\alpha\beta}{1-\theta})\theta}{(1+\alpha x)^2}(1-\alpha)\delta\eta A]x - \frac{b\eta A}{x}. \tag{A4}$$

From (A4) we can show that

$$\left(\frac{d\Delta_S}{dx} - \frac{d\mathrm{K}_S}{dx}\right)x \to -\infty \quad \text{as} \quad x \to 0 \quad \text{and} \tag{A5a}$$

$$\left(\frac{d\Delta_S}{dx} - \frac{d\mathrm{K}_S}{dx}\right)x \to +\infty \quad \text{as} \quad x \to \infty. \tag{A5b}$$

Here it is noteworthy that inequality $\mathrm{K}_S[\equiv K_{t+1}/K_t] \geq 0$ must hold true in steady states if they exist. We assume this condition in (A2). This means that there might exist an upper bound for x (e.g., Yakita, 2008). Therefore, we have $dx_{t+1}/dx_t < 1$ for sufficiently small x, and $dx_{t+1}/dx_t > 1$ for sufficiently large x.

A2 Proof of Proposition 1

Because equation $p(x;\theta) = q(x;\theta,\beta)$, we have

$$\frac{dx}{d\theta} = \frac{\partial p/\partial\theta - \partial q/\partial\theta}{\partial q/\partial x - \partial p/\partial x}. \tag{A6}$$

From (12) and (13), we obtain

$$\frac{\partial q}{\partial x} - \frac{\partial p}{\partial x} = -(1-\alpha)\delta\frac{\alpha\eta A(1+b-g) + \theta(1+\frac{\alpha\beta\eta A}{1-\theta})}{(1+\alpha x)^2} - [(1-\theta) - \frac{b\eta A}{x^2}], \tag{A7}$$

$$\frac{\partial p}{\partial\theta} - \frac{\partial q}{\partial\theta} = -(1+x) + \frac{(1-\alpha)\delta x}{1+\alpha x}[1 + \frac{\beta\alpha\eta A}{(1-\theta)^2}]. \tag{A8}$$

From the stability condition, (A7) takes a positive sign in a stable steady state. It takes a negative sign in an unstable steady state. The sign of (A8) is ambiguous; it depends on parameters. We consider two cases as in the text: (i) θ is sufficiently small and close to zero; (ii) θ is sufficiently great and close to one.

In case (i), the right-hand side of (A8) becomes

$$-(1+x)+\frac{(1-\alpha)\delta x}{1+\alpha x}(1+\beta r), \quad \text{(A9)}$$

where we use (3). Defining $\Lambda(x)\equiv-(1+x)(1+\alpha x)+(1-\alpha)\delta(1+\beta r)x$, and minimizing with respect to x, we obtain the maximum value $\bar{x}$ as

$$\bar{x}=[(1-\alpha)\delta(1+\beta r)-(1+\alpha)]/2\alpha. \quad \text{(A10)}$$

If $\beta<(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$, then $\bar{x}<0$. Because $\Lambda(0)=-1$, we have $\partial p/\partial\theta-\partial q/\partial\theta<0$ for any $x>0$ when $\beta<(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$. By contrast, if $\beta>(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$, then we have $\bar{x}>0$. In this case, if $\Lambda(\bar{x})<0$, then we have also $\partial p/\partial\theta-\partial q/\partial\theta<0$ for any $x>0$. However, if $\Lambda(\bar{x})>0$, then we have three ranges of x for $x>0$; we have $\partial p/\partial\theta-\partial q/\partial\theta<0$, $\partial p/\partial\theta-\partial q/\partial\theta>0$, and $\partial p/\partial\theta-\partial q/\partial\theta<0$ in turn. If $\beta=(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$, then we might have $\partial p/\partial\theta-\partial q/\partial\theta=0$; hence $dx/d\theta=0$. These cases are presented in Figure A1.

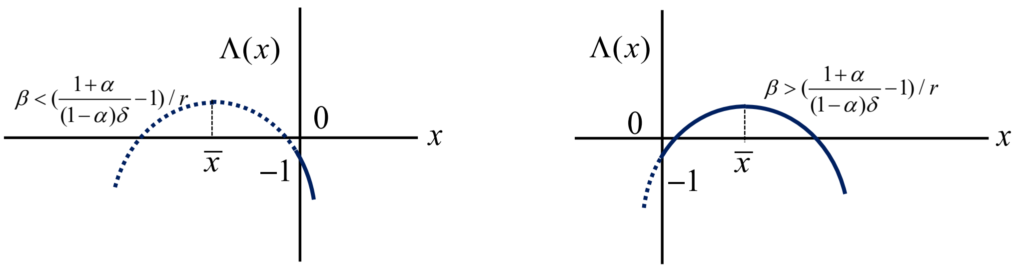

Figure A1. Debt repudiation costs.

At this stage of the argument, it is noteworthy that condition $\beta>(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$ is apparently implausible. A common value of α used in the literature is about 0.3; and the savings rate is regarded as less than 0.3. The numerator of the right-hand side of the condition is positive. Assuming an annual interest rate of 0.04 for 30 years, we have $r=2.24$. Therefore, we can assume safely that $\beta<(\frac{1+\alpha}{(1-\alpha)\delta}-1)/r$.

In case (ii), the right-hand side of (A8) becomes positive irrespective of the magnitude of β because the last term goes to positive infinity as $\theta\to0$.

Consequently, we have the following Lemma.

Lemma 1

(i) When the initial repudiation ratio is close to zero, i.e., $\theta = 0$ *, then we have* $dx^s / d\theta < 0$ *in a stable steady state and* $dx^u / d\theta > 0$ *in an unstable steady state.*

(ii) When the initial repudiation ratio θ *is sufficiently high, then we have* $dx^s / d\theta > 0$ *in a stable steady state and* $dx^u / d\theta < 0$ *in an unstable steady state.*

However, we cannot completely rule out the possibility that condition $\beta > (\frac{1+\alpha}{(1-\alpha)\delta} - 1) / r$ holds for other parameters.

Declaration of Conflicting Interests

The author declares no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Data availability Statement

Data sharing is inapplicable to this paper because no new data were analyzed.

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