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Abstract

This paper shows that, irrespective of whether free entry and exit are allowed or not, there might be a range of moderate degree of pollution emission per unit of goods within which the optimal environment tax rate is higher than the marginal environmental damage, i.e., the optimal tax rate is higher than the "Pigouvian tax rate," when the pollution function of firms and the social environmental damage function differ in their responses to a pollutant per unit of goods. However, outside the range of pollution emission, the tax rate is lower than the marginal environmental damage.

Keywords: optimal environmental policy, oligopoly, pollution emission function, product differentiation
JEL Classification: H21, H23, L13, Q58

Declaration of Conflicting Interests

The authors declare that they have no potential conflict of interest with respect to the research, authorship, or publication of this article.

Data Availability Statement

Data sharing is inapplicable to this paper because no new data were analyzed.

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1. Introduction

This paper presents examination of the optimal environmental policy in oligopolistic markets with differentiated goods. The optimal pollution tax in a competitive industry is well known to be equal to the marginal damage inflicted by the pollution (Pigou 1932). It has also been shown that the optimal pollution tax on a monopoly is less than the marginal damage (Buchanan 1969; Barnett 1980). Since the publishing of these works, many authors have investigated the optimal environmental policy in imperfectly competitive markets.[1] In such market structures, the economy has two inefficiencies: underproduction caused by imperfect competition of oligopoly and externalities caused by environmental pollution.

Simpson (1995) reports that, in a Cournot duopoly, if firms have different production costs, then the optimal tax rate might exceed the marginal damage because the tax can be an effective instrument for allocating production from the less to the more efficient firm. Lahiri and Ono (2007) report that, in an oligopolistic market, an emission standard is welfare-superior but emission-inferior to an emission tax when the number of firms is fixed. However, with free entry and exit, the results might be reversed. Fujiwara (2009) reports that optimal environmental taxation differs in the short term, with a fixed number of differentiated firms, from those in the long term, which have free entry and exit. Lian et al. (2018) demonstrate that, in both oligopoly market cases of no entry and of free entry of firms, the tax rates are smaller than the marginal environmental damage, irrespective of the degree of pollution in goods production, although assuming a nonlinear polluting technology of firms.

However, most reports of the literature assume that the pollution emission technology of firms is linear in the degree of pollution emission, e.g., a one-to-one correspondence between output and pollutant. Therefore, our research question is whether the environmental tax rate is greater than, equal to, or smaller than the marginal environmental damage when it is not linear. We call the square of a pollutant per unit of differentiated goods the degree of pollution emission. A salient feature of this paper is that it distinguishes the pollution (nonlinear) function of firms from the social environmental damage (linear) function in the assessment of pollution emission.[2] Lian et al. (2018) also introduce such a distinction in an oligopolistic market structure.[3] However, they do not fully explore the effects on optimal environmental taxation.

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[1] The optimal environmental policy in oligopolistic markets has been analyzed by many researchers, e.g., Katsoulacos and Xepapadeas (1995), Damanis (1996) and Yin (2003).

[2] We do not assume consumers' environmental awareness for expositional simplicity.

[3] In contrast, Fujiwara (2009) assumes that production of a unit of goods emits a unit of pollution as social damage.

The main results are the following. Both in the short-term and the long-term symmetric oligopolistic equilibrium, the optimal environmental policy depends on the pollution emission per unit of differentiated goods. Both in the short term and in the long term, although the optimal environmental tax rate is lower than the marginal environmental damage at higher degrees of pollution emission, the tax rate might become higher than the marginal social damage when the per-unit pollution emission becomes lower. When the degree of pollution emission becomes even lower, the optimal tax becomes lower. The effect of the difference between pollution emission and the environmental damage on the optimal environmental policy has not been analyzed formally.

A model of differentiated oligopoly is introduced in the next section. Section 3 analyzes the optimal environmental tax policy in the short term, i.e., with a fixed number of firms, and the policy in the long term, i.e., with free entry and exit. Section 4 concludes the paper.

2. Model

We consider an oligopolistic market with horizontally differentiated and homogeneous goods. Both are produced from labor. The homogeneous goods are produced with a unitary input coefficient so that the wage rate is unity, i.e., numeraire goods. The variety of the differentiated goods is designated as $n > 1$. All firms in the differentiated goods industry have the same production technology. Output of x_i units of the differentiated good requires $cx_i + f$ units of labor $(i = 1,...,n)$, where $c > 0$ and $f > 0$ respectively represent the marginal variable and fixed labor input. Following Lian et al. (2018), one unit of differentiated goods emits $\sqrt{\theta}$ units of pollution, where $\theta \in (0,1]$ is called the degree of pollution emission herein.[4] Total pollution of the industry is given as $Z = \sum_i^n e_i$, where $e_i = \sqrt{\theta} x_i [\equiv P(x_i;\theta)]$ is a pollution function. One unit of pollution is levied a pollution tax at a rate $0 < \tau < 1$. The profit of each firm is given as

$$\pi_i = (p_i - c)x_i - \tau e_i - f. \tag{1}$$

The utility function of a representative consumer is assumed as

$$u = \alpha \sum_i^n x_i - \frac{\beta - \gamma}{2} \sum_i^n (x_i)^2 - \frac{\gamma}{2} \left(\sum_i^n x_i\right)^2 + M - \frac{s}{2} Z^2. \tag{2}$$

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[4] Fujiwara (2009) assumes that each unit of differentiated goods emits one unit of pollution, i.e., $\theta = 1$.

Parameter α is the utility weight on total consumption of the differentiated goods. Also, β and γ respectively represent the preferences for product variety and for product differentiation. As commonly assumed in reports of the literature, we assume that $\beta > \gamma$. Variable M denotes consumption of homogeneous goods other than the differentiated goods. The social damage of the total pollution emission Z is given in a quadratic form: $-sZ^2/2$, where s measures the strength of damages from pollution.

Utility maximization with respect to a differentiated good, subject to the budget constraint, engenders the following inverse demand function:

$$p_i = \alpha - (\beta - \gamma)x_i - \gamma \sum_i^n x_i \quad (i = 1, ..., n). \tag{3}$$

Now we assume a symmetric equilibrium in the oligopolistic market, i.e., $x_i = x_j = x$. In the symmetric equilibrium, profits of each firm can be written as $\pi_i = [\alpha - c - \tau\sqrt{\theta} - (\beta - \gamma)x_i - \gamma \sum_j^n x_j]x_i - f$. The first-order condition for profit maximization of firms is given as

$$\frac{d\pi_i}{dx_i} = \alpha - c - \tau\sqrt{\theta} - [2\beta + \gamma(n-1)]x = 0, \tag{4}$$

from which we obtain the output-consumption level of each differentiated goods as

$$x = \frac{\alpha - c - \tau\sqrt{\theta}}{2\beta + \gamma(n-1)}. \tag{5}$$

3. Optimal environmental policies in short-term and long-term equilibrium

This section presents an analysis of the effects of the degree of pollution in producing differentiated goods on the optimal environmental policy, i.e., the optimal environmental tax rate relative to the marginal environmental damage. We first consider the short-term equilibrium with a fixed number of firms and then long-term equilibrium with free entry and exit of firms.

3.1. Short term with a fixed number of firms

In the short term, the number of firms is constant at $n > 1$. Following Lian et al. (2018), the objective function of the government is assumed as

$$W = CS + n\pi - GED + T,$$

where CS represents the consumer surplus and where GED designates the social environmental damage caused by pollution. The consumer surplus is defined as $CS = (\alpha nx - \frac{\beta - \gamma}{2} nx^2 - \frac{\gamma}{2} n^2 x^2) - pnx$. Under a symmetric equilibrium, because each firm emits $e = \sqrt{\theta} x$ units of pollution, the total pollution emitted is given as $Z = ne$. Following Fujiwara (2009) and Lian et al. (2018), the social environmental damage is defined by a quadratic form of the total pollution, i.e., $GED = sZ^2 / 2 = s\theta (nx)^2 / 2$. The tax revenue is $T = \tau en = \tau \sqrt{\theta} xn$. Therefore, the objective function can be rewritten as

$$W^N = \alpha nx - \frac{\beta - \gamma}{2} nx^2 - \frac{\gamma}{2} n^2 x^2 - pnx + n[p - c - \tau\sqrt{\theta}]x - nf$$

$$- \frac{s\theta}{2}(nx)^2 + \tau n \sqrt{\theta} x$$

$$= nx[\alpha - c - \frac{\beta - \gamma + (s\theta + \gamma)n}{2} x] - nf. \quad (6)$$

The optimal environmental tax rate τ^N is chosen to maximize the objective function (6). The first-order condition for the maximization is given as[5]

$$\frac{1}{n}\frac{dW^N}{d\tau} = \{\alpha - c - [\beta - \gamma + (s\theta + \gamma)n]x\}\frac{dx}{d\tau} = 0, \quad (7)$$

from which we obtain the optimal environmental tax rate as

$$\tau^N = \frac{(\alpha - c)(ns\theta - \beta)}{\sqrt{\theta}[\beta - \gamma + (\gamma + s\theta)n]}. \quad (8)$$

Variables with superscript N denote the values in short-term equilibrium with no entry or exit of firms. When $\theta = 1$, the optimal tax rate coincides with the one obtained in Fujiwara (2009), i.e., $\tau^N > 0$ if $ns - \beta > 0$. In this case in which the number of firms is constant, the environmental tax reduces a firm's profits. Because the tax revenue is added to the social objective, these merely offset one another. Therefore, the environmental tax affects the social objective only through changes in the output levels of differentiated goods. We assume here that $ns - \beta > 0$, as a benchmark case.[6]

The marginal environmental damage is defined as an increase in the environmental damage brought about by an additional unit of differentiated goods,

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[5] The second-order condition is satisfied, i.e., $\frac{d^2W}{d\tau^2} = -\frac{\theta n[\beta - \gamma + (\gamma + s\theta)n]}{[2\beta + \gamma(n-1)]^2} < 0$.

[6] The condition ensures a positive optimal tax rate, as described by Fujiwara (2009).

$dGED / dX = s\theta(nx)$, where $X = nx$. Letting MED denote the marginal environmental damage, we obtain the following from (5)

$$MED^N = \frac{ns\theta(\alpha - c - \tau\sqrt{\theta})}{2\beta + \gamma(n-1)}. \tag{9}$$

To analyze the properties of the optimal policy in the presence of inefficiencies caused by pollution externality and imperfect competition, we examine the sign of $\tau - MED$. If it is equal to zero, then the tax is a Pigouvian tax, merely internalizing the external diseconomies. If it has a positive sign, then it is optimal for the tax to depress the production of differentiated goods more than internalizing the external diseconomies. By contrast, if it is negative, then the tax should internalize the externalities only partially, encouraging more goods production and consumption.

From (8) and (9) we obtain

$$\tau^N - MED^N = \frac{(\alpha - c)[ns\theta(1-\sqrt{\theta}) - \beta]}{\sqrt{\theta}[\beta - \gamma + (\gamma + s\theta)n]}. \tag{10}$$

Equation (10) shows that the sign of $\tau - MED$ depends on the degree of pollution emission θ. Lian et al. (2018) report that the optimal tax rate is lower than the marginal environmental damage for all $\theta \in (0,1)$ when the tax rate is determined after firms enter the oligopolistic market. We also have $\tau^N - MED^N = -\beta(\alpha - c) / [\beta - \gamma + n(\gamma + s)] < 0$ when $\theta = 1$, as in Fujiwara (2009). Therefore, the question is whether $\tau^N - MED^N < 0$ for any $\theta \in (0,1)$.[7] From (8), we obtain

$$\frac{d\tau^N}{d\theta} = (\alpha - c)\frac{(sn + \beta/\theta)[\beta - \gamma + (\gamma + s\theta)n]/2 + sn(\beta - sn\theta)}{\sqrt{\theta}[\beta - \gamma + (\gamma + s\theta)n]^2}. \tag{11}$$

The sign of $d\tau^N / d\theta$ is indecisive *a priori*. However, we can assume that it is plausibly positive, i.e., $d\tau^N / d\theta > 0$. From (9), we also have

$$\frac{dMED^N}{d\theta} = \frac{MED^N}{\theta} - \frac{ns\tau^N\sqrt{\theta}}{2\beta + \gamma(n-1)}\left(\frac{1}{2} + \frac{\theta}{\tau^N}\frac{d\tau^N}{d\theta}\right). \tag{12}$$

The sign of the right-hand side of (12) is ambiguous, depending on the relative

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[7] We have $\tau^N - MED^N \to -\infty < 0$ as $\theta \to 0$.

magnitudes of the two terms. If the elasticity of the tax rate with respect to the degree of pollution emission $(d\tau^N / d\theta)/(\theta / \tau^N)$ is sufficiently great, then $dMED^N / d\theta$ has a small value relative to $d\tau^N / d\theta$. An increase in pollutants per unit of differentiated goods requires a higher optimal environmental tax rate because it involves a greater pollution emission per differentiated-good due to the concavity of pollution function with respect to the degree of pollution emission. The tax increase decreases the marginal environmental damage by depressing the marginal environmental damage. The high tax rate internalizes the environmental costs rather than retaining output levels per firm. In this case, we might have $\frac{d}{d\theta}(\tau^N - MED^N) > 0$. This means that we might have $\tau^N - MED^N > 0$ for some $\theta \in (0,1)$ even if $\tau^N - MED^N < 0$ when $\theta = 1$ and $\theta \approx 0$.

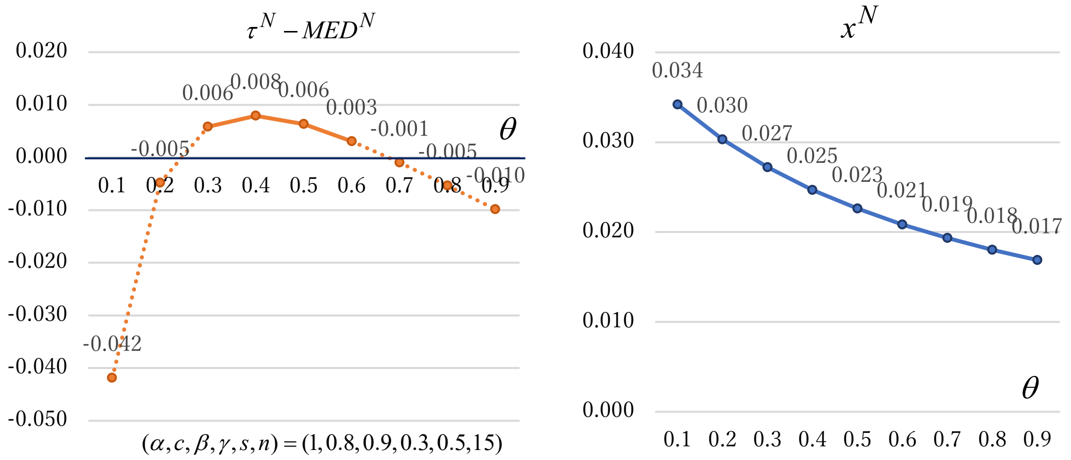
It is apparently difficult to solve algebraically. Therefore, we consider a numerical example. For our purposes, we assume an oligopoly market with 15 firms, i.e., $n = 15$. Other parameters are set as $(\alpha, c, \beta, \gamma, s, n) = (1, 0.8, 0.9, 0.3, 0.5, 15)$. The dependence of the optimal policy ($\tau^N - MED^N$) and output level (x^N) on the degree of pollution emission in production of differentiated goods is shown in Table 1 and Fig. 1. For large degrees of pollution emission, the optimal environmental tax rate is lower than the marginal environmental damage. As the degree of pollution emission becomes smaller, i.e., for $\theta \in [0.3, 0.6]$, the optimal tax rate becomes higher than the marginal environmental damage. However, if the degree of pollution emission becomes even smaller, then the optimal tax rate becomes lower than the marginal environmental damage.

Table 1 Short-term output level, environmental tax rate, and optimal policy

θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
x^N	0.034	0.030	0.027	0.025	0.023	0.021	0.019	0.018	0.017
τ^N	-0.016	0.041	0.067	0.082	0.091	0.097	0.010	0.103	0.104
$\tau^N - MED^N$	-0.042	-0.005	0.006	0.008	0.006	0.003	-0.001	-0.005	-0.010

These results can be interpreted intuitively as the following. The pollution emission function is a concave function of the degree of pollution emission. In the intermediate degree of pollution emission, the pollution emission per unit of goods is high. Therefore, it is optimal to set the tax rate higher, thereby greatly depressing the pollution emission level. With a constant number of firms, changes in the tax rate only affect the profit levels of firms. The social benefit from raising the tax revenue is offset by decreasing profits of firms (i.e., the producers' surplus).

Figure 1 Optimal policy and output level for degrees of pollution emissions in a duopoly



Therefore, we have the following proposition.[8]

Proposition 1. *Assume that pollution emissions per unit of goods be a concave function of the degree of pollution emission* $\theta \in (0,1]$ *and the social environmental damage be a quadratic function of the total pollution emissions. In an oligopolistic market with a sufficient number of symmetric firms, the second-best optimal environmental tax rate might be higher than the marginal environmental damage at an intermediate degree of pollution emission per output of differentiated goods.*

At intermediate degrees of pollution emission, the optimal environmental tax might be set as more than internalizing the environmental externality. Although this result is obtained in a numerical example, it runs in contrast to that presented by Lian et al. (2018).

It is noteworthy that the tax rate might always be lower than the marginal

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[8] From (10) we might obtain the relation between the number of firms and the degree of pollution emission, which brings about the negative sign to $\tau - MED$. It is sufficient for our purposes to report an example case of a positive sign.

environmental damage for $\theta \in (0,1]$ when the number of firms is fixed as sufficiently small.[9] When the firms are few, the inefficiency caused by imperfect competition is great. Therefore, it might be optimal to induce firms to produce more by a lower tax rate.

3.2. Long-term policy with free entry and exit

In this subsection, firms choose whether to enter (or exit), and if entering, how many goods to produce. Because firms enter the oligopolistic market to the extent that they expect positive profits, the profit of each firm becomes zero in equilibrium. Government chooses the tax rate to maximize the social objective, given these firms' reactive behaviors.

The first-order condition for the firm's profit maximization is given as (4) as in the preceding subsection. The level of output of each firm is given by (5). The maximized profit is obtained by inserting the output level in (5) into the profit function $\pi = [\alpha - c - \tau\sqrt{\theta} - (\beta - \gamma)x - \gamma nx]x - f$ as

$$\pi = \beta[\frac{\alpha - c - \tau\sqrt{\theta}}{2\beta + \gamma(n-1)}]^2 - f. \tag{13}$$

From the zero-profit condition, the equilibrium number of firms is obtained as

$$n^E = \frac{(\alpha - c - \tau\sqrt{\theta})\sqrt{\beta / f} - (2\beta - \gamma)}{\gamma}. \tag{14}$$

Variables with superscript E denote the values in the long-term equilibrium with free entry and exit. Inserting n^E from (14) into (4), one obtains the optimal output level of the differentiated goods per firm as

$$x^E = \frac{\alpha - c - \tau\sqrt{\theta}}{2\beta + \gamma(n^E - 1)} = \sqrt{\frac{f}{\beta}}. \tag{15}$$

The output level is independent of the tax rate and the number of firms.

As in the previous case, the objective function of the government is

$$W^E = \alpha nx - \frac{\beta - \gamma}{2}nx^2 - \frac{\gamma}{2}n^2x^2 - pnx - \frac{s}{2}\theta n^2x^2 + \tau\sqrt{\theta}nx,$$

where firm's profits are zero, i.e., $\pi^E = 0$. From (14) and (15), it can be rewritten as

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[9] In our numerical example, when the number of firms is less than 12, we have always $\tau^N - MED^N < 0$ for all $\theta \in (0,1)$.

$$W^E = n^E x^E [\frac{\beta-\gamma+n^E(\gamma-s\theta)}{2} x^E + \tau^E \sqrt{\theta})] . \tag{16}$$

By differentiating (16) with respect to τ, we obtain the optimal tax rate as[10]

$$\tau^E = \frac{2[(\alpha-c)/x^E-(2\beta-\gamma)]s\theta-\gamma(\beta-\gamma)}{2\sqrt{\theta}(\gamma+s\theta)} x^E . \tag{17}$$

Setting $\theta = 1$ engenders the optimal tax rate, which is obtained by Fujiwara (2009).[11] Taking a case of the positive tax rate at $\theta = 1$ as a benchmark, we analyze whether the optimal policy makes the optimal tax rate lower than the marginal environmental damage as the degree becomes lower.

The marginal environmental damage MED^E is given from (14) as

$$MED^E = s\theta \frac{(\alpha-c-\tau\sqrt{\theta})-(2\beta-\gamma)x^E}{\gamma} . \tag{18}$$

Therefore, the optimal environmental policy is given as

$$\tau^E - MED^E = \tau^E \frac{\gamma+s\theta\sqrt{\theta}}{\gamma} - s\theta \frac{\alpha-c-(2\beta-\gamma)x^E}{\gamma} , \tag{19}$$

where τ^E and x^E are given respectively by (17) and (15). When $\theta = 1$, we have $\tau^E - MED^E = -(\beta-\gamma)/2 < 0$. However, the sign of $\tau^N - MED^N$ also depends on the degree of pollution emission in the long-term case. From (17) we obtain

$$\frac{d\tau^E}{d\theta} = \frac{x}{2} \frac{2[(\alpha-c)/x^E-(2\beta-\gamma)]\frac{s\sqrt{\theta}}{2}(\gamma-s\theta)+\gamma(\beta-\gamma)[\frac{\gamma+s\theta}{2\sqrt{\theta}}+s\sqrt{\theta}]}{\sqrt{\theta}(\gamma+s\theta)^2} . \tag{20}$$

If $\gamma - s \geq 0$, then we have $d\tau^E / d\theta > 0$. However, in this case, we also assume that $d\tau^E / d\theta > 0$ for all $\theta \in (0,1)$. From (18) we also have

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[10] The second-order condition is satisfied, $d^2W / d\tau = -\theta(\gamma+s\theta)/\gamma^2 < 0$.

[11] A sufficient condition for a positive tax rate when $\theta = 1$ is given by Fujiwara (2009, p. 244).

$$\frac{dMED^E}{d\theta}=\frac{MED^E}{\theta}-\frac{s\tau\sqrt{\theta}}{\gamma}(\frac{1}{2}+\frac{\theta}{\tau^E}\frac{d\tau^E}{d\theta}). \tag{21}$$

If the elasticity of the tax rate with respect to the degree of pollution emission is sufficiently great, then we have a small value of $dMED^E / d\theta$, relative to the value of $d\tau^E / d\theta$. With a high pollution elasticity of tax rate, increases in the degree of pollution emission engenders a higher environmental tax rate because of the concavity of pollution function with respect to the degree of pollution emission. The high tax rate depresses the number of firms, mitigating the marginal environmental damage. The optimal environmental policy mitigates environmental degradation rather than reserving the output levels. In the long term, because the output level per form is constant, the policy depresses the number of firms. Therefore, in such a case, we might have $\tau^N - MED^N > 0$ for some $\theta \in (0,1)$ even if $\tau^N - MED^N < 0$ when $\theta = 1$ and $\theta \approx 0$.

As in the previous case, we cannot derive their mutual relation algebraically. Therefore, as in the previous case, we also consider a numerical example in which the parameter vector is set as $(\alpha, c, \beta, \gamma, s, f) = (1, 0.8, 0.9, 0.3, 0.5, 0.003)$. From (15), we have $x^E = 0.058$. The results for optimal policy are presented in Table 2 and Figure 2.

As the extent of pollution emission θ diminishes, the optimal tax rate becomes lower. A lower tax affects firms' expected profits positively, inducing entry of firms. Consequently, the number of firms increases. Pollution emission function of firms are a concave function of the degree of pollution emissions. The welfare damage is a linear function of total pollution emissions. Therefore, these differences make $\tau^E - MED^E$ positive: about $\theta = 0.2$ to $\theta = 0.5$. Outside this range of the degree of pollution emission, setting the environment tax rate above the marginal environmental damage is optimal. In other words, it is optimal to set the environmental tax rate higher than full internalization of pollution externalities because a negative effect of increased pollution damage brought about by firm entries overwhelms the positive effect of increased output and consumption.

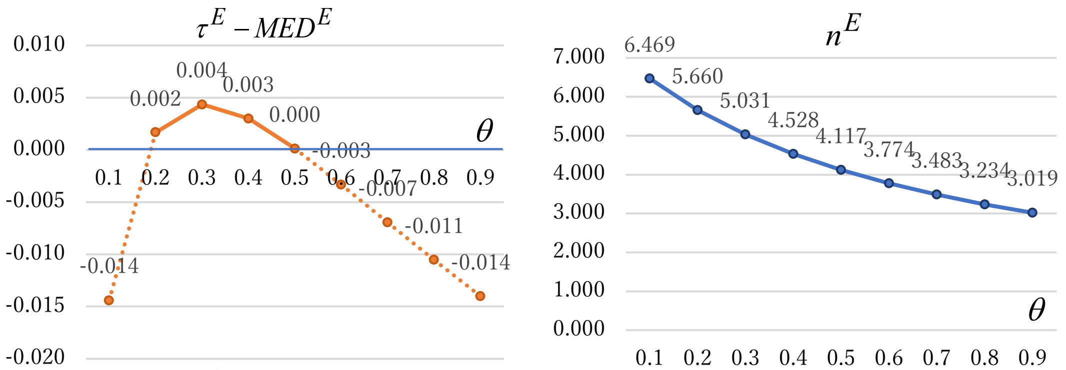
Therefore, we obtain the following proposition.

Proposition 2. *Presume that pollution emissions per unit of goods be a concave function* *of the degree of pollution emission* $\theta \in (0,1]$ *and the social environmental damage be a quadratic function of the total pollution emissions. In a symmetric oligopolistic market equilibrium with free entry and exit of firms, the second-best optimal environmental tax rate might be higher than the marginal environmental damage when the degree of pollution emission in the differentiated good production is intermediate.*

Table 2 Long-term number of firms, tax rate, and degree of pollution emissions

θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
n^E	6.469	5.660	5.031	4.528	4.117	3.774	3.483	3.234	3.019
τ^E	0.004	0.034	0.048	0.055	0.060	0.062	0.063	0.064	0.064
$\tau^E - MED^E$	-0.014	0.002	0.004	0.003	0.000	-0.003	-0.007	-0.011	-0.014

Fig. 2 Long-term optimal policy and internalization of external diseconomies.



$(\alpha, c, \beta, \gamma, s, f) = (1, 0.8, 0.9, 0.3, 0.5, 0.003)$

These results differ from those obtained by Lian et al. (2018), who analyze this free-entry-exit case as the *ex-ante* taxation case. They do not analyze the dependence of the optimal policy on the degree of pollution emission. No report in the literature describes such a study.

4. Conclusion

Results show that, irrespective of whether entry and exit of firms is allowed or not, the optimal environmental policy depends on the degree of pollution emission or pollution emission per unit of differentiated goods. In both short-term and long-term symmetric oligopolistic equilibrium, there might be a range of moderate degree of pollution emission per output of differentiated goods in which it is optimal for the optimal environmental tax rate to be set higher than the marginal environmental damage, i.e., the optimal tax rate is higher than Pigouvian tax rate, which merely internalizes the cost of external diseconomies.

The results are robust. The pollution function of firms and the social environmental damage function generally differ in their responses to the degree of pollution emission. For a moderate degree of environmental pollution, environmental taxation works to alleviate the efficiency caused by imperfect competition rather than internalizing the cost of external diseconomies. By contrast, when the degree of pollution emission is either sufficiently small or when it is sufficiently high, the optimal tax rate is lower than the marginal environment damage. The environmental taxation internalizes the cost of external diseconomies only partially, inducing differentiated goods production. Although the total output is adjusted in terms of the per-firm level of production in the short term, the number of firms adjusts the total output by entry and exit in the long term.

The simple model in this note has some weak points that must be enhanced and extended. The results are provided only in numerical examples, but not algebraically. Consumers' environmental awareness is not considered. Environmental awareness is expected to exert a positive effect on environmental policy (e.g., Yakita and Yamauchi 2011; Lian et al. 2018). These are interesting and important subjects to be analyzed in future research.

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Supplementary note

The environmental tax rate is set in response to the degree of pollution emission whereas the marginal environmental damage is proportional to the degree of pollution emission (although the out put level changes). If the concavity of pollution function is severe, then that of the corresponding tax rate is also severe. The relation between these are presented in the Fig. S1.

Fig. S1 Tax rate and marginal environmental damage
(long-term case)