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Abstract

This article analyzes the leading mergers that give insiders the strategic advantage of becoming a Stackelberg leader in free-entry markets. It is shown that mergers enhance welfare if and only if they are profitable. It is also shown that the welfare-deteriorating leading mergers do not appear in equilibrium if the entry costs are sufficiently large.

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1. Introduction

How is merger profitability affected by market structure? This issue has been extensively discussed in the growing literature on horizontal mergers. Salant et al. (1983) showed that mergers are likely to be harmful to insiders under Cournot competition unless they involve a vast majority of industry participants. However, this result depends crucially on competition structures (or the order of firms' moves). Using a Stackelberg model, Huck et al. (2004) showed that the merger between a leader and a follower is profitable regardless of the number of outsiders. Heywood and McGinty (2008) also proved the robustness of their results after considering convex costs.

As Davidson and Mukherjee (2007) point out, competitiveness (or the number of firms) is also important in merger profitability. They demonstrated that horizontal mergers among incumbent firms are always profitable in free-entry markets when both incumbent firms and entrants incur fixed costs of production. The result might differ if entry costs are incurred. Davidson and Mukherjee (2007) have also shown that merger profitability relies on the degree of cost synergy that arises from mergers. In particular, mergers with dramatic cost synergy are beneficial for insiders.

As shown in the studies above, competition structures and competitiveness have an influence on merger profitability. However, to the best of our knowledge, no studies have ever examined the influence of both these factors at the same time. The main purpose of this study is to address how the endogenous market structure, which is induced by horizontal mergers, influences merger profitability. Our study is most similar to that of Liu and Wang (2015). In their model, the merger, or the *leading merger* in their terminology, allowed insiders to enjoy the strategic advantage of becoming a Stackelberg leader. They found that the leading merger was always profitable when the number of firms is exogenous. We built off their study by intruding free entry and endogenizing the order of firms' moves and the number of firms to examine how competition structures and competitiveness affect merger profitability. Our results show that the leading merger does not benefit if the entry costs are sufficiently large, which is in sharp contrast to the findings of Liu and Wang (2015). Furthermore, our studies also show that profitable leading mergers are always welfare-enhancing.

2. Model

Consider a homogeneous-good market with infinite potential entrants, which can enter the market by incurring a fixed cost $f > 0$. In the market, there are two groups of firms (groups I and O), the members of which are indexed by $S_I = \{1, 2, \cdots, n_I\}$ ($n_I \geq 2$) and $S_O = \{n_I + 1, n_I + 2, \cdots, n_I + n_O\}$, and are producing at a constant marginal cost $c \geq 0$. Group I is a set of incumbent firms that are weighing the option to merge, whereas group O is a set of entrants in the market. It is assumed that no entrants merge. Following previous studies, we refer to the members of group I as *insiders* and those of group O as *outsiders*.

Let the output of firm $i \in S_I \cup S_O$ be denoted by q_i. Likewise, the total output of group $k \in \{I, O\}$ is denoted by q_k, that is, $q_k = \sum_{i \in S_k} q_i$. The inverse demand function for the good is given by $P(q)$, where $q = q_I + q_O$ is the industry output. The following assumption is made:

Assumption 1. *There exists a positive real number* $\bar{q}$ *such that* $P(q) = 0$ *if* $q \geq \bar{q}$ *and* $P(q) > 0$ *otherwise. Furthermore, for any* $q \in [0, \bar{q})$, $P(q)$ *is three-times continuously differentiable with* $P'(q) < 0$.

The profit of firm i is given by

$$\Pi^i(q_i, q_{-i}, f) = \begin{cases} [P(q) - c]q_i & if\ i \in S_I, \\ [P(q) - c]q_i - f & if\ i \in S_O, \end{cases}$$

where $q_{-i} = \sum_{j \neq i} q_j$. The profits of groups I and O are $\Pi^I(q_I, q_O) \equiv \sum_{i \in S_I} \Pi^i(q_i, q_{-i}, f)$ and $\Pi^O(q_O, q_I, n_O, f) \equiv \sum_{i \in S_O} \Pi^i(q_i, q_{-i}, f)$, respectively. The welfare is

$$W(q, n_O, f) = \int_0^q P(z)dz - cq - n_O f.$$

We analyze leading mergers using the following game. In stage 1, insiders decide on whether to merge. Once they agree to the merger, all insiders are integrated into one firm with one plant. In stage 2, potential entrants decide on whether to enter the market. As in Liu and Wang (2015), we assume that the merger gives insiders a strategic advantage in this game. To materialize it, we assume that stage 2 is followed by different subsequent subgames in accordance with the insiders' decisions in stage 1. If the merger is agreed upon, the merged firm commits to its output as a Stackelberg leader, while the outsiders choose their outputs as followers after observing the decision by the merged firm. If the merger falls through, both insiders and outsiders engage in Cournot competition. For convenience, we refer to a subgame consisting of both stage 2 and its subsequent subgame with Cournot (res. Stackelberg) competition as the *Cournot* (res. *Stackelberg*) *game*.

To solve for the subgame perfect Nash equilibrium (SPNE), the usual method begins by characterizing the equilibrium of the output-setting stage before proceeding to the entry stage. However, we deal with both stages simultaneously in this study. For the preparation, we provided some definitions and assumptions.

Definition. *For* $k, k' \in \{I, O\}$ $(k \neq k')$,[1]

(a) $q_k = R^k(q_{k'}, n_k)$ *such that*

$$n_k P(R^k(q_{k'}, n_k) + q_{k'}) + P'(R^k(q_{k'}, n_k) + q_{k'})R^k(q_{k'}, n_k) - n_k c = 0,$$

(b) $\tilde{\Pi}^I(q_I, n_O) \equiv \Pi^I\big(q_I, R^O(q_I, n_O)\big)$,

(c) $G(q_I, q_O, n_O) \equiv \Pi_1^I(q_I, q_O) + \Pi_2^I(q_I, q_O)R_1^O(q_I, n_O)$, *and*

(d) $\tilde{Q}^I(n_O) = \operatorname{argmax}_{q_I \in [0,\tilde{q})} \tilde{\Pi}^I(q_I, n_O)$.

Assumption 2. $\Pi_{12}^i(q_i, q_{-i}, f) = P'(q) + P''(q)q_i$ *for any* $q \in (0, \bar{q})$ *and* $q_i \in [0, q)$.

Assumption 3. G *satisfies the following:*

[1] We use subscripts to indicate the partial derivatives of multivariable functions. For instance, $\Pi_3^i(q_i, q_{-i}, f) = (\partial/\partial f)\Pi^i(q_i, q_{-i}, f)$ and $\Pi_{12}^i(q_i, q_{-i}, f) = (\partial^2/\partial q_{-i}\partial q_i)\Pi^i(q_i, q_{-i}, f)$.

(a) $G_1(q_I, q_O, n_O) < 0,\ G_2(q_I, q_O, n_O) < 0,$ *and* $|G_1(q_I, q_O, n_O)| > |G_2(q_I, q_O, n_O)|$, *and*

(b) $G_2(q_I, q_O, n_O)R_2^O(q_I, n_O) + G_3(q_I, q_O, n_O) < 0$ *and* $G_3(q_I, q_O, n_O) \geq 0$.

R^k is the aggregate reaction function of group $k \in \{I, O\}$. $\tilde{\Pi}^I$ and $\tilde{Q}^I$ are the merged firm's objective function and its profit-maximizing output respectively. Note that $G(q_I, R^O(q_I, n_O), n_O) = \tilde{\Pi}_1^I(q_I, n_O)$. This equation allows us to express the first-order condition of the merged firm by $G(q_I, R^O(q_I, n_O), n_O) = 0$. Assumption 2 ensures the second-order condition $\Pi_{11}^i(q_i, q_{-i}, f) < 0$ and the strategic substitution $R_1^k(q_{k'}, n_k) \in (-1,0)$ when outsiders exist in the market (i.e., $n_O \geq 1$). Assumption 3 (a), which is borrowed from Gal-Or (1985), ensures that the second-order condition of the Stackelberg leader holds and that the Stackelberg equilibrium is stable. Assumption 3 (b) is a natural assumption that the marginal profit will decrease as the number of entrants increase.

The equilibrium of the Cournot game is characterized by the optimality conditions and the zero-profit condition, that is, $q_k = R^k(q_{k'}, n_k)$ ($k, k' \in \{I, O\}, k \neq k'$) and $\Pi^O(q_O, q_I, n_O, f) = 0$. Likewise, the equilibrium of the Stackelberg game is characterized by $G(q_I, q_O, n_O) = 0,\ q_O = R^O(q_I, n_O)$, and $\Pi^O(q_O, q_I, n_O, f) = 0$. Let the former (res. latter) equilibrium be denoted by $(q_I, q_O, n_O) = \big(Q^{I*}(f), Q^{O*}(f), N^{O*}(f)\big)$ (res. $\big(Q^{I**}(f), Q^{O**}(f), N^{O**}(f)\big)$). As observed, the Cournot and Stackelberg games share the same conditions $F^O(q_O, q_I, n_O) = \Pi^O(q_O, q_I, n_O, f) = 0$. Shedding light on this, let the solution to the equation system be denoted by $(q_O, n_O) = \big(H^q(q_I, f), H^n(q_I, f)\big)$.

Lemma 1. *From the implicit function theorem, it follows that* $H_1^q(q_I, f) = -1,\ H_1^n(q_I, f) < 0,\ H_2^q(q_I, f) < 0,$ *and* $H_2^n(q_I, f) < 0$.

The inequalities $H_1^n < 0$, $H_2^q < 0$, and $H_2^n < 0$ are so intuitive that it requires no explanation. More interestingly, $H_1^q = -1$. This equality implies that the industry output is invariable regardless of whether the insiders become a Stackelberg leader through a leading merger. This prominent feature is a key factor in the main results.

3. Results

After the preliminaries, we now present our main results in order.

Proposition 1. *When there is free entry by outsiders, the following are equivalent:*

(a) *The leading merger is profitable.*

(b) $Q^{I*}(f) < Q^{I**}(f)$.

(c) $N^{O*}(f) > N^{O**}(f)$.

(d) $W(Q^{I*}(f) + Q^{O*}(f), N^{O*}(f), f) < W(Q^{I**}(f) + Q^{O**}(f), N^{O**}(f), f)$.

Proof: Invoking Lemma 1 (a), let $Q(f)$ be the equilibrium industry output of both the Cournot and Stackelberg games. It is apparent that for $\Pi^{I*}(f) \equiv \Pi^{I}\big(Q^{I*}(f), Q^{O*}(f)\big)$ and $\Pi^{I**}(f) \equiv \Pi^{I}\big(Q^{I**}(f), Q^{O**}(f)\big)$, there holds

$$\begin{aligned}\Pi^{I**}(f) > \Pi^{I*}(f) &\Leftrightarrow \big[P\big(Q(f)\big) - c\big]Q^{I**}(f) > \big[P\big(Q(f)\big) - c\big]Q^{I*}(f),\\ &\Leftrightarrow Q^{I**}(f) > Q^{I*}(f).\end{aligned}$$

Equivalence to (b) and (c) comes from Propositions 1 (a) and 1 (a). ■

Proposition 1 suggests that a profitable merger is always welfare-improving. This counterintuitive result is explained by the excess entry. Given that the industry output is invariant, merger profitability is determined only by market share. If the merger gives a higher market share to the insiders, the outsiders should reduce their outputs due to strategic substitution, thus confronting higher average costs. This encourages outsiders to exit the market. Accordingly, by alleviating excess entry, the leading merger improves welfare.

Proposition 1 indicates that socially undesirable mergers do not occur because such mergers are not profitable. This does not mean that insiders necessarily disagree with the merger in the SPNE. Unfortunately, our general model prevents us from deriving the necessary and sufficient conditions for insiders to disagree with the leading merger. Nevertheless, we can propose a sufficient condition.

Proposition 2. *Suppose that in the Stackelberg equilibrium with one leader and one follower, the follower earns a positive profit. The insiders never merge in the SPNE if f is sufficiently large.*

Proof: Consider $\tilde{f}$ such that $N^{O*}(\tilde{f}) = 1$. Since $\Pi^O\big(R^O(q_I, 1), q_I, 1, \tilde{f}\big)$ is decreasing in q_I, the inequality,

$$\Pi^O\big(R^O(\tilde{Q}^I(1), 1), \tilde{Q}^I(1), 1, \tilde{f}\big) > \Pi^O\big(R^O(Q^{I*}(\tilde{f}), 1), Q^{I*}(\tilde{f}), 1, \tilde{f}\big) = 0,$$

implies that $\tilde{Q}^I(1) < Q^{I*}(\tilde{f})$. Next, we prove that $\tilde{Q}^I(1) > Q^{I**}(\tilde{f})$. Define $\Phi(q_I) \equiv H^q(q_I, \tilde{f}) - R^O\left(q_I, \tilde{Q}^{I-1}(q_I)\right)$, where $\tilde{Q}^{I-1}$ is the inverse function of $\tilde{Q}^I$. Using this definition, we obtain $\Phi\left(Q^{I**}(\tilde{f})\right) = 0$. It follows from the definition of H^q and the presumption in Proposition 2 that $0 = P\left(\tilde{Q}^I(1) + H^q(\tilde{Q}^I(1), \tilde{f})\right) - c < P\left(\tilde{Q}^I(1) + R^O(\tilde{Q}^I(1), 1)\right) - c$. Accordingly, we obtained $\Phi\left(\tilde{Q}^I(1)\right) > 0$. Moreover,

$$\begin{aligned}\Phi'(q_I) &= -1 + \frac{G_1 R_2^O - G_3 R_1^O}{G_2 R_2^O + G_3} \\ &> -1 + \frac{G_2 R_2^O + G_3}{G_2 R_2^O + G_3} \quad \text{by Assumption 3} \\ &= 0\end{aligned}$$

Thus, by the intermediate value theorem, we have $\tilde{Q}^I(1) > Q^{I**}(\tilde{f})$, which yields $Q^{I*}(\tilde{f}) > Q^{I**}(\tilde{f})$. This inequality holds for large entry costs close to $\tilde{f}$ by continuity of $Q^{I*}(\tilde{f})$ and $Q^{I**}(\tilde{f})$. Therefore, using Proposition 1, we can conclude that the merger is not profitable with large entry costs. ■

The intuition behind Proposition 2 is as follows. The large entry costs discourage outsiders from entering the market under Cournot competition. Suppose then that the number of outsiders remains the same even after the leading merger forms. By the presumption of Proposition 2, positive profits induce outsiders to enter the market. This works to substitute the output of the merged firm with the output of the entrants. Consequently, the merged firm loses its market share, and thus its profit is reduced. Again, this comes from the fact that industry output is invariant before and after the leading merger.

We conclude this article by making two remarks on Proposition 2.

Remark 1. As suggested in Proposition 2, it is possible that the leading merger is not profitable. This is in sharp contrast with Liu and Wang's (2015) result that the merger is always profitable. This contrast may be emphasized in the linear demand model with $P(q) = a - q$. Straightforward computation shows that $\Pi^{I*}(f) = n_I f$ and $\Pi^{I**}(f) = (a-c)\sqrt{f}/2$, which results in

$$\Pi^{I*}(f) \gtreqless \Pi^{I**}(f) \Leftrightarrow f \gtreqless \bar{f} \equiv \frac{(a-c)^2}{4n_I^2}.$$

This implies that there is a unique entry cost $\bar{f}$ such that the leading merger is profitable if $f < \bar{f}$, but not profitable otherwise.

Remark 2. Davison and Mukherjee (2007) considered horizontal mergers in the presence of free entry. They found that mergers with dramatic cost synergy would reduce the number of firms. On the other hand, Remark 1 combined with Proposition 1 suggests that profitable leading mergers can decrease the number of firms, even if there are no cost synergy effects.

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