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Abstract

To examine the effects of a capital-use tax and automation subsidy on the proportion of automated industries and the inequality between workers and capitalists, we formulate a Schumpeterian growth model with automation, innovation, and human capital accumulation. We analytically show that a higher capital-use tax drives down the wage rate and stimulates automation on the balanced growth path, whereas a reduction in automation subsidies lowers the wage rate but inhibits automation. Moreover, we conduct a quantitative analysis demonstrating that policies that inhibit automation do indeed reduce wages; however, they can nevertheless address the disparity between workers and capitalists.

Keywords : Automation; Human capital; Inequality; Schumpeterian growth
JEL Classification: E20; O31; O38

1 Introduction

As shown in Acemoglu and Restrepo (2020), the proportion of industries in which production is based on the use of capital (machines or robots) instead of labor have rapidly increased since the advent of artificial intelligence. Furthermore, Acemoglu and Restrepo (2018, 2020)

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show that automation deprives routine workers of jobs. This implies that advances in automation may reduce the income of workers and widen the inequality between some workers and capitalists. Hence, policies that promote automation lead to greater inequality. If this is correct, the question arises: can policies that inhibit automation address the inequality?

To answer this question, we construct a Schumpeterian growth model with the following features. First, there are two types of households: workers who accumulate human capital and supply their labor, and capitalists who accumulate physical capital. Second, the intermediate goods production industry can be divided into two categories: automated industries in which physical capital is used as an input, and nonautomated industries in which labor is used. Finally, R&D activities for innovation and automation coexist. In terms of policies aimed at inhibiting automation, we consider an increase in a capital-use tax[1] and a decrease in R&D subsidies for automation.

We show that the balanced growth path (BGP) uniquely exists if the productivity of physical capital is relatively larger than that of labor in the production of intermediate goods.[2] By focusing on the BGP, we obtain the following results. First, policies aimed at inhibiting automation decrease the share of automated industries; however, they also decrease the wage rate. This result implies that policies aimed at reducing inequality between workers and capitalists could worsen inequality by decreasing the wage income of workers. To consider whether such policies worsen inequality, we conduct a quantitative analysis using US data. We show that they increase the worker's consumption relative to capitalist's consumption and, thus, the utility gap between workers and capitalists decreases.

There are several papers that examine automation in the context of economic growth models.[3] Acemoglu and Restrepo (2018) construct a horizontal R&D model with automation in which the production of older types of goods become obsolete when new products are developed. Acemoglu et al. (2020) introduce wage and interest income taxes into the model of Acemoglu and Restrepo (2018) and compare the present US tax code with an optimal tax rate. Gasteiger and Prettner (2022) consider an overlapping generations model in which savings are allocated into physical and robot capital; they consider the effect of taxation on the use of robot capital on the transitional dynamics of each input and its price. Hémous and Olsen (2022) construct a horizontal R&D model with automation and replicate the pattern of the US skill premium, which is increasing but has done so at a declining rate in recent years.

Our model is closely related to the study of Chu et al. (2023) in that there are both workers

[1]As physical capital is used in the automated industry in our model, we consider that a capital-use tax will inhibit automation. Some papers, such as Guerreiro et al. (2022) and Thuemmel (2023), use models in which machines or robots are used in the automated industry and consider what they refer to as a robot tax. By regarding physical capital as robot capital, a capital-use tax is essentially the same as a robot tax.

[2]This condition means that marginal cost of producing an intermediate good decreases as a result of automation. That is, there are incentives to invest in R&D activities for automation.

[3]Recent research includes Chu et al. (2020), Okada (2020), Shimizu and Momoda (2020), Klarl (2022), Momoda et al. (2022), Abeliansky and Prettner (2023), Aisa et al. (2023), Dawid and Neugart(2023), Dinopoulos et al. (2023), Ikeshita et al. (2023), Pillai (2023), Prettner (2023), Jacobs (2024), Jones and Liu (2024), and Leduc and Liu (2024).

and capitalists, and both innovation and automation R&D in their study and ours. However, there are two major differences between the two models. First, workers endogenously accumulate human capital in our model. Second, the goal of Chu et al. (2023) is to analyze the effects of subsidies for R&D activities on social welfare, whereas our study aims to examine whether policies that aim to inhibit automation address the inequality between workers and capitalists.

The remainder of this paper is organized as follows. We describe the model in Section 2, and we show the existence and uniqueness of the BGP under certain conditions in Section 3. In Section 4, we conduct comparative statics regarding the effect of each policy on the wage rate and the share of automated industries on the BGP. This is followed by a quantitative analysis to consider the effect of each policy on the disparity between workers and capitalists. Finally, Section 5 summarizes and concludes the study.

2 The model

We develop a quality-ladder model with two types of households and two types of R&D activities. Time is continuous. Households are categorized into workers or capitalists. Workers supply labor and accumulate human capital, whereas capitalists accumulate physical capital. Final good is produced by using a unit continuum of intermediate goods. There are two states of intermediate good industries, automated and nonautomated. In automated industries, firms input physical capital to produce, whereas in nonautomated industries, firms input labor. The state of the industries switches as a result of R&D activities for automation and innovation. Given the level of productivity, which improves via innovation, automation R&D changes nonautomated industries into automated ones. Only firms in nonautomated industries invest in R&D activities for automation. In contrast, R&D activities for innovation are undertaken in both industries. Regardless of their states, once R&D activities for innovation succeed, productivity improvements lead to the industries becoming nonautomated. The government imposes a labor income tax and a capital-use tax, and the government provides subsidies for R&D activities for both automation and innovation.

2.1 Households

We consider infinitely-lived households. There are two types of households, workers and capitalists. All workers are identical, and all capitalists are identical. Hereafter, variables relating to workers or nonautomated industries are characterized by the superscript h, and those relating to capitalists or automated industries are characterized by the superscript k. Each household can hold financial assets, which are the stocks of intermediate good firms. Only capitalists can hold physical capital, and only workers can accumulate human capital. The population of workers is normalized to one, whereas that of capitalists is L^k.

The representative worker allocates their effective labor h_t, which depends on their human capital level, to human capital accumulation and labor supply. A representative worker has the intertemporal utility function U^h, given as:

$$U^h = \int_0^\infty e^{-\rho t} \log c_t^h dt, \tag{1}$$

where ρ is a subjective discount rate and c_t^h is the consumption of workers per capita. The per capita flow budget constraint of workers is as:

$$\dot{a}_t^h = r_t a_t^h + (1 - \tau_t^h) w_t (h_t - e_t) - c_t^h \equiv r_t a_t^h + \tilde{w}_t (h_t - e_t) - c_t^h, \tag{2}$$

where a dot over a variable indicates a differentiation with respect to time t. In (2), a_t^h is the asset holdings of workers per capita, and r_t is the rate of return on assets. e_t represents efforts to accumulate human capital and, thus, $h_t - e_t$ is the effective labor supply per capita. Let w_t denote the wage rate per effective labor. Due to income tax, the rate of which is given by τ_t^h, workers receive only $(1 - \tau_t^h) w_t$ per unit of effective labor. Here, $\tilde{w}_t \equiv (1 - \tau_t^h) w_t$ represents an after-tax wage rate. Following Chu et al. (2013), the law of motion of human capital is as:

$$\dot{h}_t = B e_t - \delta^h h_t, \tag{3}$$

where B and δ^h are the effectiveness of efforts and the depreciation rate of human capital, respectively. The workers maximize their intertemporal utility subject to (2) and (3). From the maximum principle, we can obtain the utility-maximizing condition of workers as follows:

$$\frac{\dot{c}_t^h}{c_t^h} = r_t - \rho, \tag{4}$$

$$e_t \begin{cases} > 0 & \text{if } r_t = B - \delta^h + \dot{\tilde{w}}_t / \tilde{w}_t \\ = 0 & \text{if } r_t > B - \delta^h + \dot{\tilde{w}}_t / \tilde{w}_t \end{cases}. \tag{5}$$

Equation (4) is the familiar Euler equation. Regarding (5), $B - \delta^h + \dot{\tilde{w}}_t / \tilde{w}_t$ denotes the return on human capital accumulation. If the return on asset holdings exceeds that on human capital, the workers do not invest in the latter; otherwise, they increase their level of effort to accumulate human capital as long as these returns are equalized. Hence, when efforts are positive, the following workers' no-arbitrage condition (NAC) between asset holdings and human capital accumulation holds:

$$r_t = B - \delta^h + \dot{\tilde{w}}_t / \tilde{w}_t. \tag{6}$$

The capitalists neither supply labor nor accumulate human capital; instead, they invest some of their income into physical capital accumulation as well as financial assets holdings. A representative capitalist's income consists of the return on physical capital k_t and that on financial assets a_t^k. The capitalist has an intertemporal utility function U^k, which has the same functional form as the worker's utility function. The per capita flow budget constraint of the capitalists is such as:

$$\dot{a}_t^k + \dot{k}_t = r_t a_t^k + (R_t - \delta^k) k_t - c_t^k, \tag{7}$$

where R_t is the rental rate of physical capital, δ^k is the depreciation rate of physical capital, and c_t^k is the consumption of capitalists per capita. Similarly, we can obtain the utility-maximizing condition of capitalists such as:

$$\frac{\dot{c}_t^k}{c_t^k} = r_t - \rho, \tag{8}$$

$$r_t = R_t - \delta^k, \tag{9}$$

where (9) represents the capitalists' NAC between financial assets and physical capital.

2.2 Final good

The final good market is perfectly competitive. The price of final goods is normalized to one, that is, final goods are the numéraire. Final good Y_t is produced by using a unit continuum of intermediate goods, which is indexed by $i \in [0, 1]$. The production function is given as:

$$Y_t = \exp\left(\int_0^1 \log x_t(i) di\right), \tag{10}$$

where $x_t(i)$ represents intermediate goods produced in industry i. By defining $p_t(i)$ as the price of $x_t(i)$, the profit-maximizing condition gives the conditional demand function of $x_t(i)$ as follows:

$$x_t(i) = \frac{Y_t}{p_t(i)}. \tag{11}$$

2.3 Intermediate good

A unit continuum of industries, indexed by $i \in [0, 1]$, is divided into two groups depending on whether the industrial state is automated or nonautomated. The state of industries switches as a result of R&D activities for automation and innovation. If nonautomated industry succeeds with its R&D for automation, that industry switches and becomes automated. If innovation occurs in an industry, that industry becomes nonautomated with the productivity improvement regardless of its previous state.[4] Following Chu et al. (2023), we assume that each industry is a monopoly because other incumbent firms exit due to cost disadvantages. Following Zeira (1998), a firm in an automated industry uses physical capital to produce intermediate goods, and a firm in a nonautomated industry uses labor according to the following respective production functions:

$$x_t(i) = \begin{cases} \frac{1}{Z_t} z^{n_t(i)} \ell_t(i) & \text{if nonautomated,} \\ \frac{A}{Z_t} z^{n_t(i)} k_t(i) & \text{if automated,} \end{cases} \tag{12}$$

[4] As in Acemoglu and Restrepo (2018) and Chu et al (2023), we assume that " humans have a comparative advantage in new and more complex tasks ".

where $z > 1$ is the common step size, $n_t(i)$ is the number of innovations that have occurred in industry i as of time t, $\ell_t(i)$ and $k_t(i)$ are labor and physical capital inputs, respectively, in industry i, and $A > 0$ is a parameter that captures an exogenous productivity difference between automated and nonautomated industries. Z_t is an aggregator of quality improvement defined as:

$$Z_t \equiv \exp\left(\int_0^1 n_t(i)di \log z\right), \tag{13}$$

which negatively affects the productivity of each input. This represents the negative externality whereby more factor inputs are needed to produce intermediate goods in an economy where high quality goods have already been produced. If the firms use physical capital, they have to pay a capital-use tax, which is an ad valorem tax, set at rate τ^k. Then, the profit of firms that produce $x_t(i)$ is as follows:

$$\pi_t(i) = \begin{cases} p_t(i)x_t(i) - w_t\ell_t(i) & \text{if nonautomated} \\ p_t(i)x_t(i) - (1+\tau^k)R_t k_t(i) & \text{if automated} \end{cases}. \tag{14}$$

We assume that the markup ratio is exogenously given by $\mu > 1$ following Evans et al. (2003) and Chu et al. (2023). Then, the monopolistic firm charges the following price:

$$p_t(i) = \begin{cases} \mu \dfrac{Z_t w_t}{z^{n_t(i)}} & \text{if nonautomated} \\ \mu \dfrac{Z_t(1+\tau^k)R_t}{Az^{n_t(i)}} & \text{if automated} \end{cases}. \tag{15}$$

Once prices, such as (15), are determined, substituting (11), (12), and (15) into (14) yields

$$\pi_t(i) = \pi_t = \frac{\mu - 1}{\mu} Y_t, \tag{16}$$

which does not depend on the industries and their states.

Defining $\theta_t \in [0, 1]$ as a share of automated industries, let Θ_t and $\mathcal{N}_t$ denote the set of automated and nonautomated industries, respectively. Then, using (11), (12), and (15), the aggregate demands for labor and physical capital, L_t and K_t, respectively, are given as follows:

$$L_t \equiv \int_{i\in\mathcal{N}_t} \ell_t(i)di = (1-\theta_t)\frac{Y_t}{\mu w_t}, \tag{17}$$

$$K_t \equiv \int_{i\in\Theta_t} k_t(i)di = \theta_t \frac{Y_t}{\mu(1+\tau^k)R_t}. \tag{18}$$

2.4 Innovation and automation

Competitive entrepreneurs engage in R&D activities for innovation and automation by hiring labor, and both R&D activities are subsidized by the government.

For entrepreneurs to have incentives to engage in automation, we must assume that automation decreases the marginal cost of producing intermediate goods. This condition is given as follows:

$$\frac{Z_t w_t}{z^{n_t(i)}} > \frac{Z_t(1+\tau^k)R_t}{Az^{n_t(i)}}.$$

Similarly, for entrepreneurs to have incentives to engage in innovation, the step size of the innovation must be large enough that the marginal cost of production becomes lower via innovation. Considering the setting that successful innovation makes the industry nonautomated, the condition is given as follows:

$$\frac{Z_t(1+\tau^k)R_t}{Az^{n_t(i)}} > \frac{Z_t w_t}{z^{n_t(i)+1}}.$$

To satisfy the above two conditions, we focus on the following parameter ranges:

$$1 < \frac{Aw_t}{(1+\tau^k)R_t} < z. \tag{19}$$

Innovation for industry $i \in [0, 1]$ succeeds with a Poisson arrival rate of $\lambda_t(i)$ given as:

$$\lambda_t(i) = \tilde{\varphi}_t h_t^h(i), \tag{20}$$

where $h_t^h(i)$ is the labor input for innovation in industry i, and $\tilde{\varphi}_t$ is the productivity of innovation. There is a negative externality in the sense that R&D for innovation becomes more difficult as the economy develops. This externality is specified as:

$$\tilde{\varphi}_t \equiv \frac{\varphi}{Y_t}, \tag{21}$$

where φ is a positive parameter. The expected profit from innovation in industry i, $\pi_t^I(i)$, is:

$$\pi_t^I(i) = \lambda_t(i)v_t^h(i) - (1-s)w_t h_t^h(i),$$

where $v_t^h(i)$ is the value of innovation in industry i and s is a subsidy rate for innovation. From the free-entry condition for innovation, $\pi_t^I(i) = 0$, v_t^h becomes

$$v_t^h = \frac{(1-s)w_t Y_t}{\varphi}. \tag{22}$$

Note that we omit index i because the right-hand side of (22) does not depend on i, and the values of innovation are symmetric among industries.

Automation for the nonautomated industry $i \in \mathcal{N}_t$ succeeds with a Poisson arrival rate, $\alpha_t(i)$, given as:

$$\alpha_t(i) = \tilde{\phi}_t h_t^k(i), \tag{23}$$

where $h_t^k(i)$ is the labor input for automation in industry i, and $\tilde{\phi}_t$ is the productivity of automation. There are two negative externalities of R&D for automation. One is an externality similar to that for innovation. The other captures the feature that industries that are difficult to automate are likely to remain nonautomated and, thus, a share of automated industries θ negatively affects the productivity of automation. Those externalities are specified as:

$$\tilde{\phi}_t \equiv \frac{(1-\theta_t)\phi}{Y_t}. \tag{24}$$

where ϕ is a positive parameter. The expected profit of automation in industry i, $\pi_t^A(i)$, is:

$$\pi_t^A(i) = \alpha_t(i)v_t^k(i) - (1-\sigma)w_t h_t^k(i)$$

where $v_t^k(i)$ is the value of automation in industry i, and σ is the subsidy rate for automation. From the free-entry condition for automation, $v_t^k(i)$ becomes

$$v_t^k = \frac{(1-\sigma)w_t Y_t}{\phi(1-\theta_t)}. \tag{25}$$

We omit index i for the same reason as explained above.

The NACs for innovation and automation, respectively, are:

$$r_t = \frac{\pi_t + \dot{v}_t^h - (\lambda_t + \alpha_t)v_t^h}{v_t^h}, \tag{26}$$

$$r_t = \frac{\pi_t + \dot{v}_t^k - \lambda_t v_t^k}{v_t^k}. \tag{27}$$

The term π_t in (26) and (27) denotes dividends, $\dot{v}_t^h$ and $\dot{v}_t^k$ are capital gains, and $(\lambda_t + \alpha_t)v_t^h$ and $\lambda_t v_t^k$ are capital losses due to creative destruction. Because innovation will occur in all industries, whereas automation will occur only in nonautomated industries, the probabilities of creative destruction are different. Note that we have already omitted index i from λ_t and α_t in (26) and (27) because π_t, v_t^h, and v_t^k do not depend on i. [5]

The share of automated industries θ_t increases by automation and decreases by innovation. Hence, we have the following dynamics of θ_t

$$\dot{\theta}_t = \alpha_t(1-\theta_t) - \lambda_t\theta_t. \tag{28}$$

In (28), the first term represents the inflow of automated industries, whereas the second term represents the outflow toward nonautomated industries.

[5]From (27), it is evident that λ_t does not depend on i. Thus, from (26), we can show that α_t does not depend on i.

2.5 Government

The government adopts a balanced budget at each point of time. It raises tax revenue from workers via a labor income tax and from firms in automated industries via a capital-use tax. The tax revenue is used for two types of R&D subsidies. The flow budget constraint of the government is given as follows:

$$\tau_t^h w_t(h_t - e_t) + \tau^k R_t K_t = s w_t h_t^h + \sigma(1-\theta_t) w_t h_t^k. \tag{29}$$

2.6 Market-clearing conditions

In this economy, there are five markets: the labor, physical capital, final good, intermediate goods, and financial assets markets. We have already imposed market-clearing conditions for intermediate goods implicitly.

An effective labor supply, $h_t - e_t$, is demanded for the production of intermediate goods by nonautomated industries $L_t = (1-\theta_t)Y_t/(\mu w_t)$, for innovation h_t^h, and for automation in nonautomated industries, $(1-\theta_t)h_t^k$. The labor market–clearing condition is given as:

$$h_t - e_t = (1-\theta_t)\frac{Y_t}{\mu w_t} + h_t^h + (1-\theta_t)h_t^k. \tag{30}$$

The physical capital market-clearing condition is given as:

$$L^k k_t = K_t. \tag{31}$$

Final goods are used for consumption and the formation of physical capital. Then, the final good market-clearing condition is as:

$$Y_t = C_t + \dot{K}_t + \delta^k K_t, \tag{32}$$

where $C_t \equiv L^k c_t^k + c_t^h$ represents aggregate consumption. The financial asset market-clearing condition is given as:

$$a_t = (1-\theta_t)v_t^h + \theta_t v_t^k, \tag{33}$$

where $a_t \equiv L^k a_t^k + a_t^h$ represents aggregate asset holdings. [6]

[6]A flow financial assets market-clearing condition is given by

$$\dot{a}_t - r_t a_t = \frac{d}{dt}\left[(1-\theta_t)v_t^h + \theta_t v_t^k\right] - r_t\left[(1-\theta_t)v_t^h + \theta_t v_t^k\right].$$

By assuming the initial financial assets market-clearing condition $(1-\theta_0)v_0^h + \theta_0 v_0^k = a_0$, we can obtain (33).

3 Balanced growth path

In this section, we define a BGP and demonstrate its existence and uniqueness. First, we define the BGP as follows.

Definition 1. *The BGP is a path that satisfies the following features.*

1. *All of Y_t, C_t, a_t, K_t, h_t, v_t^h, v_t^k, and e_t grow at the same rate given by g. That is,*

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{a}_t}{a_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{h}_t}{h_t} = \frac{\dot{v}_t^h}{v_t^h} = \frac{\dot{v}_t^k}{v_t^k} = \frac{\dot{e}_t}{e_t} = g. \tag{34}$$

2. *$\theta, \tilde{w}, R$, and r are constant over time. That is,*

$$\dot{\theta}_t = \dot{\tilde{w}}_t = \dot{R}_t = \dot{r}_t = 0. \tag{35}$$

Hereafter, we focus on the BGP.

Proposition 1. *Let us assume that*

$$\frac{A}{(1+\tau^k)\mu R} > 1, \tag{36}$$

where $R = B - \delta^h + \delta^k$. Then, there uniquely exists the BGP with $\theta \in (0,1)$ and $g = B - \delta^h - \rho$, which is characterized by the following two equations:

$$w = \frac{\mu - 1}{\mu}\rho\left(\frac{\phi}{1-\sigma} - \frac{\varphi}{1-s}\right)\frac{1-\theta}{\theta} \equiv \omega(\theta; s, \sigma), \tag{37}$$

$$w = \left(\frac{1}{\mu}\right)^{\frac{1}{1-\theta}}\left[\frac{A}{(1+\tau^k)R}\right]^{\frac{\theta}{1-\theta}} \equiv \Omega(\theta; \tau^k). \tag{38}$$

Proof. *See Appendix A.*

□

Proposition 1 states that the solution of (37) and (38) characterizes the BGP, and our model exhibits semi-endogenous growth.[7] Equation (37) is derived from the stationary condition for the share of automated industries, that is, from (28), with $\dot{\theta} = 0$, and the NACs for

[7]We restrict the parameter regions that the solution of (37) and (38) must satisfy.

$$1 < \frac{Aw}{(1+\tau^k)R} < z, \qquad \frac{\phi(1-\theta)}{1-\sigma} < \frac{\varphi}{1-s} < \frac{\phi}{1-\sigma}$$

The former ensures the incentives for innovation and automation, and the latter guarantees that $\alpha > 0$ on the BGP.

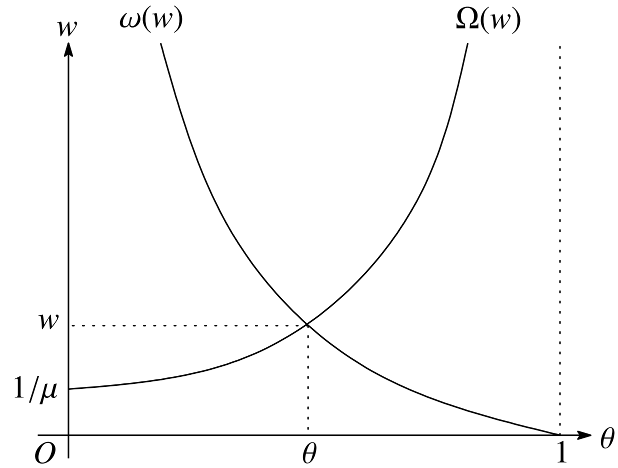


Figure 1: The determination of θ and w

innovation (26) and automation (27). Equation (37) means that there is negative relationship between θ and w. The reason for this is as follows. A decrease in w reduces the costs of both innovation and automation and raises their respective arrival rates, λ and α. Because the probability of capital loss in nonautomated industries is $\lambda + \alpha$, whereas that in automated industries is λ, a decrease in w increases the probability of capital loss in nonautomated industries relative to automated industries. Then, investing in automation becomes more attractive, leading to an increase in the share of automated industries θ.

Equation (38) indicates that the unit cost of producing the final good under optimal conditions is equal to the price of the final good, which is a numéraire. Equation (38) means that there is positive relationship between θ and w under the assumption given by (36). The assumption requires that the marginal cost of intermediate goods in automated industries used for final good production is smaller than that in nonautomated industries. This implies that the marginal cost of physical capital is smaller than that of labor in the aggregate production function. The reason why (38) is upward sloping in the (θ, w) space under (36) is as follows. If the share of automated industries θ increases, demand for physical capital increases, which leads to a lower unit cost because physical capital is inexpensive. This allows for a higher expenditure on labor inputs, making higher wages affordable while keeping the unit cost at one.

Figure 1 plots (37) and (38) in the (θ, w) space, and it shows the existence and uniqueness of the BGP with $\theta \in (0, 1)$.

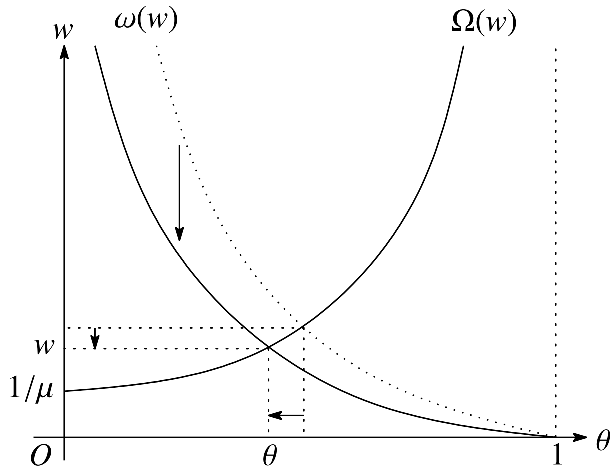


Figure 2: Comparative statics of a decrease in σ

4 Policy analysis

We conduct a policy analysis with respect to the capital-use tax τ^k and subsidies for automation σ on the BGP to show the effect of policies aimed at halting automation.[8] We analytically investigate the effects on the share of automated industries θ and the wage rate w. In addition, to obtain the effect on inequality between capitalists and workers, we calibrate our model using data from the US.

4.1 Comparative statics

First, we examine the effect of a decrease in σ on θ and w. Recall that σ only affects (37). By differentiating (37) with respect to σ, we obtain $\partial\omega/\partial\sigma > 0$. Then, we can obtain the result shown in Figure 2. We summarize the effect of σ in the following proposition:

Proposition 2. *A decrease in subsidies for automation σ reduces both the share of automated industries θ and the wage rate w on the BGP.*

As expected, a decrease in σ reduces the share of automated industries θ. Surprisingly, however, this policy change intended to expand labor demand by increasing the share of nonautomated industries leads to a decrease in the wage rate.

The intuition for Proposition 2 is as follows. Given w, if the subsidy rate for automation, σ, decreases, the cost of automation increases and the share of the automated industries θ decreases. Then, the variety of relatively inexpensive intermediate goods in the automated industries shrinks. It makes it acceptable wage less to keep a unit cost one.[9]

[8] The result of comparative statics with respect to the share of subsidies for innovation, s, is qualitatively the same as that with respect to a decrease in σ. Hence, we focus on the latter analysis.

[9] Here, the expression "acceptable wage" means the maximal wage to make a unit cost less than the price of final good given by one.

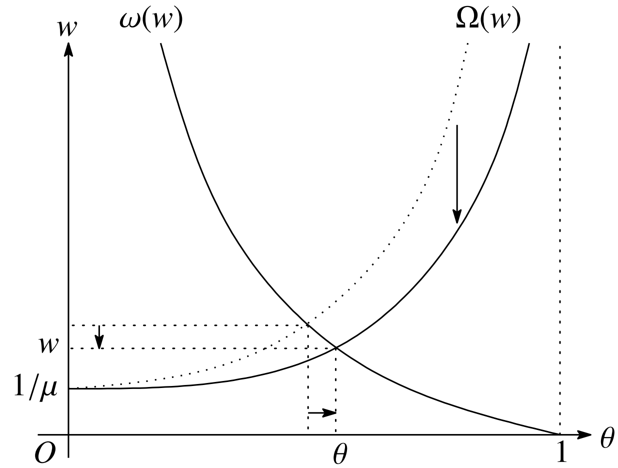


Figure 3: Comparative statics of an increase in τ^k

Second, we examine the effect of an increase in τ^k on θ and w. Note that τ^k only affects (38). By differentiating (38) with respect to τ^k, we obtain $\partial\Omega/\partial\tau^k < 0$. Then, we can obtain the result shown in Figure 3. We summarize the effect of τ^k in the following proposition:

Proposition 3. *An increase in the capital-use tax rate τ^k raises the share of automated industries θ and reduces the wage rate w on the BGP.*

The result of Proposition 3 seems counterintuitive because the aim of the increase in the capital-use tax rate τ^k is to inhibit automation and raise the wage rate. We can interpret the result of Proposition 3 as follows. Given θ, if τ^k increases, the marginal cost of intermediate goods produced by using physical capital increases in automated industries. This makes the acceptable wage less to keep unit costs at one and, thus, w decreases. Furthermore, the arrival rates of both innovation λ and automation α increase because of the decrease in the labor cost. Then, the probability of capital loss in nonautomated industries increases relative to automated industries, and investing in the nonautomated industries become less attractive, which leads to an increase in θ.

4.2 Quantitative analysis

The comparative statics results suggest that policies aimed at inhibiting automation do indeed reduce wages, which may widen the utility gap between capitalists and workers. To examine whether such a policy increases inequality, we focus on the ratio of consumption per capita of capitalists and workers,[10] and show the effect of the capital-use tax τ^k and subsidy for

[10]Because the instantaneous utility function is logarithmic and the consumption of both capitalists and workers grows at the same rate of g, the effect of each policy on the ratio of consumption per capita of capitalists and workers is qualitatively the same as the difference among their lifetime utility on the BGP; $U^k - U^h = (1/\rho)\log(c^k/c^h)$.

δ^k	ρ	μ	z	σ_b	s_b	τ_b^k	A	φ	ϕ
0.043	0.050	1.100	1.200	0.188	0.188	0.100	0.149	448.246	559.377

Table 1: Parameter values in the quantitative analysis

g_b	$(RK+\pi)/GDP$	$R\&D/GDP$	$L^k a_0^k/a_0$
0.023	0.400	0.046	0.300

Table 2: Target values in the quantitative analysis

automation σ on it quantitatively. Our analysis focuses only on the BGP, that is we assume that the economy is initially on the BGP.

We set and calibrate parameter values as shown in Table 1.[11] The value of the depreciation of physical capital δ^k, is sourced from US data for the period of 2010–2017. We follow Chu et al. (2023) in setting the values of the subjective discount rate ρ, the markup ratio μ, and the benchmark level of the subsidy for innovation s_b, and in assuming that the benchmark level of the subsidy for automation σ_b is equal to s_b. The values of the innovation step size z are adopted from Impullitti (2010). Following Acemoglu et al. (2020), we set the benchmark value of τ_b^k to 0.1.

To calibrate the remaining parameter values, we use the target values shown in Table 2. The target value of the growth rate, g_b, is the average growth rate of the US for 2010–2023. Following Chu et al. (2023), we set the capital share $(RK+\pi)/GDP$ to 0.4. Using the US data on investment in intellectual property rights for the period of 2010–2023, we estimate the R&D/GDP ratio as 0.046. Using these three target values, we calculate the benchmark level of θ_b at around 0.397. The value of A is set to satisfy (19) under the benchmark level of θ_b. Using the values of θ_b and A, from (38), we can calculate the benchmark level of w_b to be around 0.944. We can calibrate the values of φ and ϕ to satisfy (37) and the parmeter condition that guarantees α to be positive on the BGP.[12] We arbitrarily set the value of the initial ratio of capitalists' assets to total assets, $L^k a_0^k/a_0$, at 0.3, meaning that capitalists hold 30% of total assets in the initial period. Then, we can obtain the ratio of consumption per capita of capitalists to workers, c^k/c^h.[13,14,15]

Figure 4 simulates the effects of an R&D subsidy for automation σ. Figure 4(a) shows that a decrease in σ has a positive effect on the ratio of the worker's consumption per capita c^h to final good production Y. In contrast, a decrease in σ decreases the ratio of the capitalist's

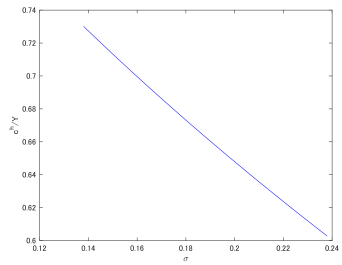
[11] Hereafter, the subscript b represents the benchmark or target values.

[12] See footnote 7.

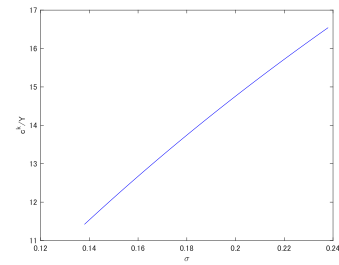
[13] See Appendix B on calibrating these parameters values.

[14] The qualitative results remain unchanged if we set the initial ratio of capitalists' assets to total assets as 0.7

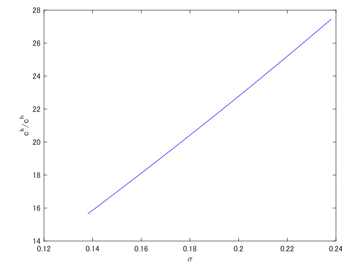
[15] These data are available from the US Bureau of Economic Analysis.



(a) The effect on c^h/Y.

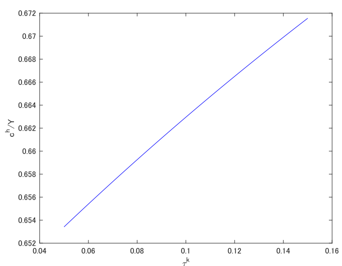


(b) The effect on c^k/Y.

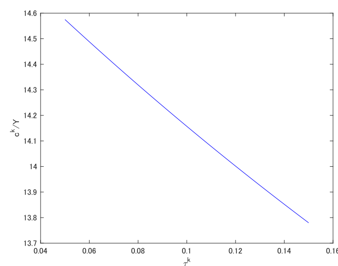


(c) The effect on c^k/c^h.

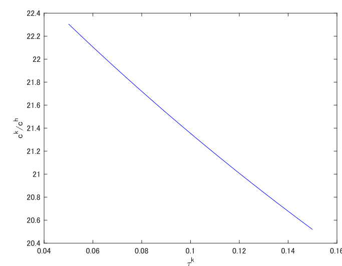
Figure 4: The effects of a change of σ



(a) The effect on c^h/Y.



(b) The effect on c^k/Y.



(c) The effect on c^k/c^h.

Figure 5: The effects of a change of τ^k

consumption per capita c^k to Y as Figure 4(b) shows.[16] Because c^h/Y increases and c^k/Y decreases as an R&D subsidy for automation decreases, the ratio of consumption per capita between a capitalist and a worker c^k/c^h decreases, as shown in Figure 4(c).

Figure 5 simulates the effects of a capital-use tax τ^k. Figure 5(a) shows that an increase in τ^k has a positive effect on the ratio of the worker's consumption per capita c^h to Y. Figure 5(b) shows that an increase in τ^k decreases the ratio of the capitalist's consumption per capita c^k to Y, contrastively. Because c^h/Y increases and c^k/Y decreases as a capital-use tax increases, the ratio of consumption per capita between a capitalist and a worker c^k/c^h decreases, as shown in Figure 5(c). Therefore, we can quantitatively show that the policies aimed at inhibiting automation do indeed address the inequality between workers and capitalists, although they do decrease the wage rate.

[16]Note that c^k is per capita variable. In Figure 4(b), c^k/Y is greater than 1, but this is not strange. This is because the aggregate capitalist consumption is c^kL^k and $L^k = 0.01$.

5 Concluding remarks

It is often stated that the advance of automation deprives workers of their jobs and increases the inequality between capitalists and workers. Therefore, in this study, we asked the question, can policies to inhibit automation address this inequality? To answer this question, we have constructed a Schumpeterian growth model with the following features. First, the model has two types of households, workers and capitalists. Second, the intermediate goods industries can be divided into two states: automated industries that use physical capital for production, and nonautomated ones that use labor. Finally, R&D activities for innovation and automation coexist. As policies designed to inhibit automation, we have considered an increase in capital-use tax and a decrease in subsidies for automation.

Initially, we demonstrated the existence and uniqueness of the BGP of our model. Then, by focusing on the BGP, we have shown that policies aimed at inhibiting automation can decrease the share of automated industries; however, these policies also decrease the wage rate. To consider whether such policies can reduce inequality, we have conducted a quantitative analysis and shown that they do indeed reduce inequality. This implies that such policies for inhibiting automation can address inequality between workers and capitalists, although they do decrease the wage rate. Hence, policy makers can address the inequality between workers and capitalists by imposing capital-use taxes and/or decreasing subsidies for automation.

Appendices

Appendix A. Proof of Proposition 1

From (4), (8), and (34), we have

$$g = r - \rho. \tag{A.1}$$

Using (6), (35), and (A.1) yields the growth rate

$$g = B - \delta^h - \rho. \tag{A.2}$$

From (3) and (A.2), the value of e_t/h_t is given by

$$\frac{e_t}{h_t} = 1 - \frac{\rho}{B}. \tag{A.3}$$

From (9) and (A.2), a rental rate of physical capital R becomes

$$R = r + \delta^k = B - \delta^h + \delta^k. \tag{A.4}$$

Hereafter, we regard R as a parameter because it is constant from (A.4). From (18), we obtain the value of K_t/Y_t such that

$$\frac{K_t}{Y_t} = \frac{\theta_t}{\mu(1+\tau^k)R}. \tag{A.5}$$

By substituting (16), (25), and (A.1) into (27), the value of λ_t on the BGP becomes:

$$\lambda_t = \frac{\mu - 1}{\mu} \frac{\phi(1 - \theta_t)}{(1 - \sigma) w_t} - \rho. \tag{A.6}$$

Similarly, from (26), the value of α_t on the BGP becomes:

$$\alpha_t = \frac{\mu - 1}{\mu w_t} \left(\frac{\varphi}{1 - s} - \frac{\phi(1 - \theta_t)}{1 - \sigma} \right). \tag{A.7}$$

From (28), (35), (A.6), and (A.7), the value of θ on the BGP is:

$$\theta_t = \frac{\alpha_t}{\lambda_t + \alpha_t} = \frac{\frac{\phi}{1-\sigma} - \frac{\varphi}{1-s}}{\frac{\mu}{\mu-1}\rho w_t + \frac{\phi}{1-\sigma} - \frac{\varphi}{1-s}}. \tag{A.8}$$

To ensure that $\theta_t > 0$ on the BGP, from (A.8), we must restrict the parameter ranges such that

$$\frac{\phi}{1 - \sigma} - \frac{\varphi}{1 - s} > 0.$$

Similarly, to ensure that $\alpha_t > 0$ on the BGP, from (A.7), we must restrict the parameter ranges such that

$$\frac{\varphi}{1 - s} > \frac{\phi(1 - \theta_t)}{1 - \sigma}.$$

Hence, to ensure that both $\theta_t > 0$ and $\alpha_t > 0$, we assume that

$$\frac{\phi}{1 - \sigma} > \frac{\varphi}{1 - s} > \frac{\phi(1 - \theta_t)}{1 - \sigma}. \tag{A.9}$$

By solving (A.8) for w_t, we can obtain the following relationship between w_t and θ_t on the BGP:

$$w_t = \frac{\mu - 1}{\mu} \rho \left(\frac{\phi}{1 - \sigma} - \frac{\varphi}{1 - s} \right) \frac{1 - \theta_t}{\theta_t} \equiv \omega(\theta_t; s, \sigma). \tag{A.10}$$

Equation (A.10) is the first equation that characterizes the BGP.

Labor demand for innovation h_t^h on the BGP is derived from (20) and (21) such as:

$$h_t^h = \frac{\lambda}{\tilde{\varphi}_t} = \frac{\lambda Y_t}{\varphi}. \tag{A.11}$$

Labor demand for automation h_t^k on the BGP is derived from (23) and (24) such as:

$$h_t^k = \frac{\alpha}{\tilde{\phi}_t} = \frac{\alpha Y_t}{(1 - \theta_t)\phi}. \tag{A.12}$$

Then, from (30) with (A.11) and (A.12), the value of Y_t/h_t becomes

$$\frac{Y_t}{h_t} = \left[\frac{1-\theta_t}{\mu w_t} + \frac{\lambda_t}{\varphi} + \frac{\alpha_t}{\phi}\right]^{-1} \frac{\rho}{B}, \tag{A.13}$$

where λ_t, α_t, and θ_t are given by (A.6), (A.7), and (A.8), respectively. By substituting (12), (17), and (18) into (10), we obtain

$$1 = \left[\frac{A}{(1+\tau^k)\mu R}\right]^{\theta_t} \left(\frac{1}{\mu w_t}\right)^{1-\theta_t}. \tag{A.14}$$

Therefore, by rewriting (A.14), we can obtain the relationship between w_t and θ_t on the BGP as:

$$w_t = \left(\frac{1}{\mu}\right)^{\frac{1}{1-\theta_t}} \left[\frac{A}{(1+\tau^k)R}\right]^{\frac{\theta_t}{1-\theta_t}} \equiv \Omega(\theta_t;\tau^k). \tag{A.15}$$

Equation (A.15) is the second equation that characterizes the BGP.

Equations (A.10) and (A.15) include only two endogenous variables, θ_t and w_t. To show the existence and uniqueness of the solution for (A.10) and (A.15), we check the properties of $\omega(\theta_t; s, \sigma)$ and $\Omega(\theta_t;\tau^k)$. For $\omega(\theta_t; s, \sigma)$, we can show that $\lim_{\theta_t\to 0}\omega(\theta_t;\tau^k) = +\infty$, $\lim_{\theta_t\to 1}\omega(\theta_t;\tau^k) = 0$, and $d\omega(\theta_t; s, \sigma)/d\theta_t < 0$. For $\Omega(\theta_t;\tau^k)$, we can show that $\lim_{\theta_t\to 0}\Omega(\theta_t;\tau^k) = 1/\mu$. Then, taking the logarithm of (A.15) and differentiating it with respect to θ_t yields as follows:

$$\frac{d\Omega(\theta_t;\tau^k)}{d\theta_t} = \frac{\Omega(\theta_t;\tau^k)}{(1-\theta_t)^2} \log\frac{A}{(1+\tau^k)\mu R}. \tag{A.16}$$

The sign of this derivative is positive if

$$\frac{A}{(1+\tau^k)\mu R} > 1. \tag{A.17}$$

By assuming (A.17), equations (A.10) and (A.15) must have only one intersection in the range of $\theta_t \in (0, 1)$. Thus, unique time-invariant inner solution of θ_t and w_t exists. This solution satisfies Definition 1 of the BGP. Hereafter, we omit the subscript t of the time-invariant variables.

Finally, by substituting (A.3), (A.4), (A.11), (A.12), and (A.13) into (29), the labor income-tax rate is given as follows:

$$\tau^h = \left[s\frac{\lambda}{\varphi} + \sigma\frac{\alpha}{\phi} - \frac{\tau^k\theta}{\mu(1+\tau^k)w}\right]\left[\frac{1-\theta}{\mu w} + \frac{\lambda}{\varphi} + \frac{\alpha}{\phi}\right]^{-1}. \tag{A.18}$$

Appendix B. Calibration of the benchmark parameters

The first half of this appendix shows how to calibrate parameters A, φ, and ϕ. To begin, from (A.2) and (A.4), the rental rate is:

$$R = g_b + \rho + \delta^k,$$

and from (9), the return rate of financial asset is given by

$$r = R - \delta^k.$$

Next, we define $\xi \equiv (RK + \pi)/GDP$, and $\chi \equiv R\&D/GDP$. As GDP consists of final goods production and investment in R&D, $GDP = Y/(1-\chi)$. By using the definition of ξ, (16), and (A.5), we can derive the benchmark value of the share of automated industries, θ_b, given as:

$$\theta_b = \left(1 - \mu + \frac{\mu}{1-\chi}\xi\right)(1 + \tau_b^k).$$

We set the value of A to satisfy (19). By substituting (A.10) into (19), it can be rewritten as:

$$\mu(1 + \tau_b^k)R < A < \mu(1 + \tau_b^k)Rz^{1-\theta_b}. \tag{B.1}$$

Then, we set A to the average of the upper and lower bounds of (B.1), that is,

$$A = \frac{\mu(1 + \tau_b^k)R(z^{1-\theta_b} + 1)}{2}.$$

From (A.15), the benchmark value of the wage rate w_b is:

$$w_b = \Omega(\theta_b; \tau_b^k) = \left(\frac{1}{\mu}\right)^{\frac{1}{1-\theta_b}} \left[\frac{A}{(1 + \tau_b^k)R}\right]^{\frac{\theta_b}{1-\theta_b}},$$

where we note that both θ_b and w_b do not depend on ϕ and φ.

In order to satisfy (A.9), we set φ to the average of the upper and lower bounds of (A.9), that is,

$$\varphi = \left(1 - \frac{\theta_b}{2}\right)\frac{1 - s_b}{1 - \sigma_b}\phi. \tag{B.2}$$

To calibrate the values of ϕ, from (A.10) and (B.2), we have

$$\phi = 2\frac{\mu}{\mu - 1}\frac{w_b}{\rho}\frac{1 - \sigma_b}{1 - \theta_b}. \tag{B.3}$$

In the latter half of this appendix, we derive the initial ratios of the total consumption of capitalists and workers to Y. Before deriving the ratios, we define $\kappa \equiv L^k a_0^k / a_0$ as the share of capitalists' asset holdings to total asset holdings in the initial period. Note that r, w, τ^h, and θ are time independent because we focus on the BGP. Then, from the flow budget constraint for the capitalist (7), as well as the transversality condition of the capitalist $\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds\right)(a_t^k + k_t) = 0$, we obtain the initial ratio of the total consumption of capitalists to Y such as:

$$L^k c_0^k = \rho(L^k a_0^k + L^k k_0) \qquad \Leftrightarrow \qquad \frac{L^k c_0^k}{Y_0} = \rho\left(\kappa \frac{a_0}{Y_0} + \frac{K_0}{Y_0}\right).$$

Similarly, from the flow budget constraint for the worker (2) and the transversality condition of the worker $\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds\right) a_t^h = 0$, the initial consumption of workers becomes

$$c_0^h = \rho(a_0^h + H),$$

where H is the sum of the discounted present value of disposable labor income, defined as

$$H \equiv \int_0^\infty \exp\left(-\int_0^t r_s ds\right) \tilde{w}_t(h_t - e_t)dt = \frac{(1-\tau^h)wh_0}{B}.$$

Then, we obtain the initial ratio of the consumption of workers to Y as:

$$c_0^h = \rho\left(a_0^h + \frac{(1-\tau^h)wh_0}{B}\right) \qquad \Leftrightarrow \qquad \frac{c_0^h}{Y_0} = \rho\left((1-\kappa)\frac{a_0}{Y_0} + \frac{(1-\tau^h)wh_0}{Y_0 B}\right).$$

For $(1-\tau^h)wh_0/(Y_0 B)$, by substituting (A.3), (A.6), (A.7), (A.11), and (A.12) into (30), we obtain

$$\frac{wh_0}{BY_0} = \frac{1}{\rho\mu}\left[1-\theta+(\mu-1)\frac{1-\theta}{1-\sigma}\left(\frac{\phi}{\varphi}-1\right)+(\mu-1)\frac{1}{1-s}\frac{\varphi}{\phi}\right] - \frac{w}{\varphi},$$

and τ^h is given by (A.18).

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