Chukyo University Institute of Economics

Discussion Paper Series

April 2012

No. 1201

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Abstract

We examine the effects of public pensions on the residential choice of a child, who is altruistic, and provides aged parents with attention as well as financial support in two ways: income transfers and contribution to family public goods. We find that, even if the child lives with parents in the same home under a certain level of public pensions, the child eventually chooses to move away from the parents as the level of public pension rises. When the child moves, both the parents' and child's welfare may decrease. Nevertheless, the optimal level of public pensions is positive under reasonable parameter values in the social welfare function.

JEL: H41, H55, J10

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1. Introduction

It has been often argued that, while social security contributes to mitigate the uncertainty in the living of the elderly, it weakens family bonds by reducing the willingness of adult children to support their aged parents. In accordance with this commonly held view, the percentage of elderly people over 65 years old living with their children has been decreasing with the development of the public pension system in Japan (see Figure 1).

In this paper, taking the residential choice of adult children into account, we study the effects of public pensions on family bonds, and examine how the family welfare is there by affected. Our model consists of a two-period game between the parents and the child in a family. In the first period, the child, who is a young adult and has been living in her parents' home, chooses her location and becomes employed in the labour market in the region where she lives. The child's future earnings depend on her location choice. She may choose to continue to live with her parents and work in the home region, or to move to another region with better earning opportunities. In the second period, the parents age and require attention (or care). The level of attention the parents receive from the child depends on the geographical distance between the parents and child. The child contributes to public pensions from her income and allocates the rest among her consumption of private goods, contribution to family public goods and income transfers toward her parents. The parents allocate the sum of their income (e.g. income from interest), public pension benefits and income transfers from her child between their consumption of private goods and contribution to family public goods.

Incorporating family public goods into a model of location choice is a unique feature of this paper. All family members living in the same home can receive benefits from family public goods, such as houses, gardens, household appliances and housework. However, such spill-over effects almost disappear after the child leaves the parental home.

We assume that neither parents nor child move in the second period because the cost is too high for professional or social reasons. Under this assumption, anticipating the outcome of the game between the parents and child in the second period, the child decides her location strategically in the first period. The significant factors in making this decision are considered to be as follows: 1) the difference in earnings among regions. The child is more likely to move away from the parents if there is a greater potential for higher earnings away from the parents' location. 2) the level of attention the child gives the parents. The further away a child lives from parents, the lower the attention level becomes. Therefore, the child's location choice depends on the child's preference for giving parents attention. 3) the difference in the cost of living in terms of the distance between the parents and child. When they live in the same home, several types of goods serve as family public goods and thus the total expenditures of the parents and child can be relatively kept down. The child has potentially two ways of financial support for the parents: income transfers and contribution to the family public goods. However, the child can do it only through income transfers if she lives away from the parents.

Based on these three factors, we first examine the condition under which the child chooses to live with her parents in the same home. Second, we analyze how the condition is affected by the level of public pensions. More precisely, supposing an initial situation where the child lives with her parents given a certain level of public pensions, we examine the effect of an increase in public pensions on the child's location choice. Third, we examine how public pensions affect the parents' and the child's welfare through the child's location choice, and also investigate the optimal level of public pensions under different assumptions on the social welfare function.

Konrad *et al*. (2002) and Rainer and Siedler (2009) study the mobility pattern of two siblings who have the responsibility of providing care for their parents. Although those studies constitute a notable precursor to our analysis, the purpose is basically different: we focus on the impact of social security on location choices and the welfare in a family, whereas social security and any other public policies are not within the scope of those studies. In addition, those studies consider solely attention or care as what children provide to their aged parents, and ignore any financial support. From our point of view, financial support such as income transfers and provision of family public goods by children also contributes to improve parental well-being, and should have an interaction with location choices in a family.

This paper is organized as follows. Section 2 describes the model. Section 3 examines the effect of public pensions on the child's location choice. In Section 4, we examine the welfare effect of public pensions and point out the possibility that an increase in public pensions is Pareto disimproving. In Section 5, we introduce the social welfare function and examine the optimal level of public pensions. Section 6 summarizes the paper.

2. Model

We consider a linear economy where the economic activity is made on the real line, and a representative family that consists of parents and a child. The parents live and raise their child at some place that is normalized to 0 and they do not move from there.

Our model consists of two periods. In the first period (in the first stage), the child chooses her location $k(\geq 0)$ soon after finishing school. She is employed in the labor market in the region she lives, and earns her income $Y(k)$ there. The child's income depends on her location and we make the following assumption: the maximum income is obtained at k^c, and the income falls as the child lives farther from k^c, where $k^c(>0)$ represents the central business district in the linear economy. This implies that, when the child lives in the same home or locality as her parents and becomes employed in the local labor market, her income becomes less than when she becomes employed in the central business district: $\underset{k}{Max}\, Y(k) = Y(k^c) > Y(0)$ with $Y'(k) \geq 0$ for $k \in [0,\ k^c]$ (equality holds when $k = k^c$) and $Y'(k) < 0$ for $k \in (k^c,\ \infty)$.

We consider two types of goods: private goods and family public goods. The benefits from family public goods spill over to all family members. We assume that, as long as family members live in the same home, the family public goods have the property of pure public goods. The supply of family public goods is thus equal to the sum of contributions made by the parents and child, g_p and g_k, if $k=0$. On the other hand, even when the parents and child do not live in the same home ($k>0$), the property of public goods still exists to some extent for several types of family public goods if they live in the same neighborhood and visit each other's home very frequently. However, such spill-over effects get smaller as the distance between parents and child becomes greater, eventually disappearing at a certain distance. Therefore, the levels of family public goods consumed by the parents and child, G_p and G_k, are determined as follows:

(1) $$G_p = g_p + \gamma(k) g_k,$$

(2) $$G_k = g_k + \gamma(k) g_p,$$

where $\gamma(k)$ indicates the magnitude of spill-over effects of the child's (parents') contribution on the parents' (child's) consumption of family public goods. It is assumed that $0 \le \gamma(k) \le 1$, $\gamma(0)=1$ for $k=0$ and $\gamma(k)=0$ for $k \ge \bar{k}(>0)$.

In the second period, the parents retire and allocate the sum of their income (e.g., income from interest) Y_p, public pension benefits T_p and income transfers from their child $\pi(\ge 0)$ between their consumption of private goods C_p and contribution to family public goods g_p. The budget constraint for the parents is thus given by

(3) $$C_p = Y_p - g_p + \pi + T_p.$$

The child contributes T_k to public pensions from her income $Y_k(k)$ and allocates the rest among her consumption of private goods C_k, contribution to family public goods g_k, and income transfers toward her parents. The budget constraint for the child is thus given by

(4) $$C_k = Y_k(k) - g_k - \pi - T_k.$$

Assuming that all families are identical, we have $T_k = T_p = T$ under a pay-as-you-go public pension system.

In the second period, the parents need attention (or care), and the level of attention the child gives to the parents depends on the distance between the parents and child because longer travel time means a greater cost of the visit. We assume that the child does not move to the parents because the cost of moving is too high for professional or social reasons, as in Konrad *et al*. (2002). This implies that the location the child has chosen in the first period determines the level of attention the parents receive in the second period. Therefore, denoting the distance between the parents and child (the distance between points 0 and k) as $\delta(0,\ k)$, the level of attention is provided as $a = a(\delta(0,\ k)) = a(k)$ with $da(k)/dk < 0$.

The child is altruistic toward her parents, and her utility function is given by

(5) $$U_k = \log C_k + \alpha \log G_k + v_k(a(k)) + \rho U_p,$$

where ρ $(0 < \rho < 1)$ is the weight attached to the parents' utility U_p, and $\alpha > 0$ is assumed. On the other hand, the parents are non-altruistic and their utility function is given by

(6) $$U_p = \log C_p + \alpha \log G_p + v_p(a(k)).$$

According to Bernhaim *et al*. (1985), we assume that both the parents' and child's utility derived from attention, $v_p(\cdot)$ and $v_k(\cdot)$, first increase and then decrease in a ($v_p'' < 0$ and $v_k'' < 0$), and that the parents' utility $v_p(\cdot)$ always increases when the child's utility $v_k(\cdot)$ does not decrease in a.

Also, we make the following assumption on $a(0)$, the level of attention when the parents and child live in the same home:

(7) $$\arg\max_a v_k(a) \le a(0) \le \arg\max_a v_k(a) + \rho v_p(a),$$

which implies that the child's *private* utility of attention is decreasing while the child's *total* utility (including the altruistic term) is increasing when $k = 0$: $v_k'(a(0)) \le 0$ and $v_k'(a(0)) + \rho v_p'(a(0)) \ge 0$. From (7), we also find that the parents' utility is increasing when $k = 0$: $v_p{}'(a(0)) \ge 0$.

From the assumptions made above, $v_k'(a(0)) + \rho v_p'(a(0)) \ge 0$, $v_k''(a(k)) + \rho v_p''(a(k)) < 0$ and $a'(k) < 0$, we have $v_k'(a(k)) + \rho v_p'(a(k)) > 0$ for $k > 0$. This implies that the child chooses her location $k \in [0,\ k^c]$. The reason is as follows. Consider two locations, $k_1 \in [0,\ k^c)$ and $k_2 \in (k^c,\ \infty)$, where the child can earn same income. The child prefers k_1 to k_2 because the farther away she lives from her parents, the less attention she gives them. It also should be noted that $Y'(k) \ge 0$ for $k \in [0,\ k^c]$.

The timing of the game in the second period is as follows: the parents choose their contribution to family public goods g_p in the second stage, and then the child chooses her consumption of private goods C_k, her contribution to family public goods g_k, and income transfers toward her parent π in the third stage. (As a result, the parents' consumption of private goods C_p is determined in the third stage.)

3. Effect of public pensions on location choice

In this section, we derive the subgame perfect equilibrium of the model presented in the previous section, and examine the effect of public pensions on the child's location choice.

In the third stage, given the parents' contribution to family public goods g_p, the location k, and the contribution to public pensions T, the child chooses the contribution to family public goods g_k and income transfers to the parents π (≥ 0) so as to maximize her utility (5). The first-order conditions for maximization are[1]

$$(8) \qquad -\frac{1}{Y_k(k)-g_k-\pi-T}+\frac{\rho}{Y_p-g_p+\pi+T}\leq 0 \text{ (equality holds if } \pi>0),$$

$$(9) \qquad -\frac{1}{Y_k(k)-g_k-\pi-T}+\frac{\alpha}{g_k+\gamma(k)g_p}+\frac{\rho\alpha\gamma(k)}{g_p+\gamma(k)g_k}=0.$$

The child's reaction functions are derived from (8) and (9), and defined as

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[1] For the following reasons, g_k always takes a positive value. First, since we consider a child supporting her parents financially, we assume away the case where both π and g_k are zero. Second, even if we consider the non-negativity constraint on g_k explicitly, it cannot take a corner solution when $\pi>0$. This is proved as follows. Suppose that $\pi>0$ and the non-negativity constraint on g_k is binding. The first-order condition with respect to g_k $(-1/(Y_k(0)-\pi-T)+\alpha(1+\rho)/g_p<0)$ and (8) with equality imply

$$\frac{-\rho}{Y_p-g_p+\pi+T}+\frac{\alpha(1+\rho)}{g_p}<0.$$

This is equal to the marginal utility of g_p (the left-hand side of (15)) because $\partial\pi/\partial g_p=1/(1+\rho)$ is obtained from (8) if $\pi>0$ and $g_k=0$. We thus have $g_p=0$. However, this implies $\alpha(1+\rho)/g_p=\infty$, which contradicts the first-order condition with respect to g_k.

(10) $$\pi = \pi(g_p,\ k,\ T) = \begin{cases} \pi^+(g_p,\ k,\ T) & \text{(if (8) holds with equality),} \\ 0 & \text{(if (8) holds with strict inequality),} \end{cases}$$

(11) $$g_k = g_k(g_p,\ k,\ T) = \begin{cases} g_k^+(g_p,\ k,\ T) & \text{(if (8) holds with equality),} \\ g_k^0(g_p,\ k,\ T) & \text{(if (8) holds with strict inequality),} \end{cases}$$

with

(12) $$\frac{\partial \pi^+}{\partial g_p} = 1 + \frac{(1-\gamma)}{D}\frac{1}{C_k^2}\left(-\frac{1}{G_k^2} + \frac{\rho\gamma}{G_p^2}\right) > 0,$$

(13) $$\frac{\partial g_k^+}{\partial g_p} = -1 + \frac{(1-\gamma)}{D}\left(\frac{1}{C_k^2} + \frac{\rho}{C_p^2}\right)\left(\frac{1}{G_k^2} - \frac{\rho\gamma}{G_p^2}\right) < 0,$$

(14) $$\frac{\partial g_k^0}{\partial g_p} = -\frac{\alpha\gamma[(1/G_k^2) + (\rho/G_p^2)]}{(1/C_k^2) + \alpha[(1/G_k^2) + (\rho\gamma^2/G_p^2)]} < 0,$$

where[2]

$$D = \frac{1}{C_k^2}\left(\frac{\alpha}{G_k^2} + \frac{\rho\alpha\gamma^2}{G_p^2}\right) + \frac{1}{C_p^2}\left(\frac{1}{C_k^2} + \frac{\alpha}{G_k^2} + \frac{\rho\alpha\gamma^2}{G_p^2}\right) > 0.$$

In the second stage, given k and T, taking the child's reaction functions (10) and (11) into account, the parents choose the contribution to family public goods g_p so as to maximize their utility (6). The first-order condition for maximization is

(15)
$$\frac{1}{Y_p - g_p + \pi + T}\left(\frac{\partial \pi^+}{\partial g^p} - 1\right) + \frac{\alpha}{g_p + \gamma(k)g_k}\left(1 + \gamma(k)\frac{\partial g_k^+}{\partial g_p}\right) \le 0 \text{ (if (8) holds with equality),}$$

(16)
$$-\frac{1}{Y_p - g_p + T} + \frac{\alpha}{g_p + \gamma(k)g_k}\left(1 + \gamma(k)\frac{\partial g_k^0}{\partial g_p}\right) \le 0 \text{ (if (8) holds with strict inequality).}$$

We define the parents' reaction function derived from (15) and (16) as

(17) $$g_p = g_p(k,\ T) = \begin{cases} g_p^+(k,\ T) & \text{(if (8) holds with equality),} \\ g_p^0(k,\ T) & \text{(if (8) holds with strict inequality).} \end{cases}$$

From (12)-(14), if the child lives together with the parents ($k = 0$), we have $\gamma = 1$,

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[2] The derivation of (12)-(14) and (18) is shown in Appendix.

and thus $\partial\pi^{+}/\partial g_{p}=1$, $\partial g_{k}^{+}/\partial g_{p}=-1$ and

(18) $$\frac{\partial g_{k}^{0}}{\partial g_{p}}=-(1-\theta)<0,$$

where

(19) $$\theta=\frac{\alpha(1+\rho)}{1+\alpha(1+\rho)}\quad(0<\theta<1).$$

This implies that, if (8) holds with equality (namely, $\pi>0$ or $\pi=0$ as the interior solution), the left-hand side of (15) is zero for any value of g_{p}, so that indeterminacy arises for g_{p}. It follows from (8) and (9) that the indeterminacy of g_{p} entails the indeterminacy of π and g_{k}. This result is similar to that obtained in Cornes, Itaya and Tanaka (2012). The following proposition provides a sufficient condition under which we have $\pi=0$ and $g_{p}=0$ as the corner solution, and the indeterminacy does not arise in the equilibrium of the subgame beginning at the second stage, given that the child lives together with the parents.

Proposition 1. Given $k=0$. If $\rho(1-\theta)<(Y_{p}+T)/(Y_{k}(0)-T)<1/\alpha$, then $\pi=0$ and $g_{p}=0$.

Proof:

Consider the child's choice on π in the third stage. The first-order condition (8) implies that, given $k=g_{p}=0$, we have $\pi=0$ if

(20) $$\rho<\frac{Y_{p}+T}{Y_{k}(0)-g_{k}-T}.$$

Substituting $k=0$ $(\gamma(0)=1)$, $g_{p}=0$ and $\pi=0$ into (9) yields

(21) $$g_{k}=\theta[Y_{k}(0)-T].$$

Substituting (21) into (20) yields

(22) $$\rho(1-\theta)<\frac{Y_{p}+T}{Y_{k}(0)-T}.$$

Given $k=g_{p}=0$, we have $\pi=0$ if (22) holds.

Next, we examine the parents' choice on g_{p} in the second stage when (22) holds.

We define $\hat{g}_p$ as the level of the parents' contribution to family public goods such that income transfers π are operative for $g_p > \hat{g}_p$.[3] We have

(23) $$\left.\frac{dU_p}{dg_p}\right|_{g_p>\hat{g}_p} = \frac{1}{Y_p - g_p + \pi + T}\left(\frac{\partial \pi^+}{\partial g^p} - 1\right) + \frac{\alpha}{g_p + g_k}\left(1 + \frac{\partial g_k^+}{\partial g_p}\right).$$

Since we have $\partial \pi^+ / \partial g_p = 1$ and $\partial g_k^+ / \partial g_p = -1$ if $k = 0$ as shown above, (23) is equal to zero. On the other hand, since $\pi = 0$ for $g_p \le \hat{g}_p$, we have

(24) $$\left.\frac{dU_p}{dg_p}\right|_{0\le g_p \le \hat{g}_p} = -\frac{1}{Y_p - g_p + T} + \frac{\alpha}{g_p + g_k}\left(1 + \frac{\partial g_k^0}{\partial g_p}\right).$$

Substituting $g_k = -(1-\theta)g_p + \theta(Y_k(0) - T)$, which is obtained from (9) with $k = 0$, and (18) into (24) yields

(25) $$\left.\frac{dU_p}{dg_p}\right|_{0\le g_p \le \hat{g}_p} = -\frac{1}{Y_p - g_p + T} + \frac{\alpha}{Y_k(0) + g_p - T},$$

which is negative,[4] if

(26) $$\frac{Y_p + T}{Y_k(0) - T} < \frac{1}{\alpha}.$$

Given $k = 0$, therefore, U_p is maximized at $g_p = 0$.

The above argument shows that, if (22) and (26) are simultaneously satisfied, we have $g_p = 0$ and $\pi = 0$. □

Proposition 1 suggests that, when the child lives with the parents in the same home, both income transfers from the child to the parents and the parents' contribution

[3] The child's marginal utility of π is increasing in g_p:

$$\frac{\partial}{\partial g_p}\left(\frac{\partial U_k}{\partial \pi}\right) = -\frac{-(-1)}{C_k^2}\frac{\partial g_k^0}{\partial g_p} + \frac{\rho}{C_p^2} = \frac{1-\theta}{C_k^2} + \frac{\rho}{C_p^2} > 0.$$

Therefore, the child chooses positive π under a sufficiently large level of g_p.

[4] Since $(Y_p - g_p + T)/(Y_k(0) + g_p - T)$ is decreasing in g_p, we have $(dU_p / dg_p)_{0\le g_p \le \hat{g}_p} < 0$ if $(Y_p + T)/(Y_k(0) - T) < 1/\alpha$.

to family public goods are zero, given reasonable parameter values. For example, under $\rho = 0.6$ and $\alpha = 1$, we have $g_p = 0$ and $\pi = 0$ if $0.6/2.6(\approx 0.23) \le (Y_p + T)/(Y_k(0) - T) < 1$. The ratio of disposable income of the retired generation to that of the working generation is likely to take a value within this range. In the analysis below, we assume that the sufficient condition in Proposition 1 is satisfied.

We now examine the child's location choice in the first stage. She chooses $k\ (\ge 0)$ so as to maximize the utility function (5) subject to the reaction functions (10), (11) and (17). Using the envelope theorem, the first-order condition is reduced to

$$\begin{aligned}\frac{dU_k}{dk} &= \frac{Y_k'(k)}{Y_k(k) - g_k - \pi - T} + \alpha \frac{\gamma'(k) g_p(k,\ T) + \gamma(k)\partial g_p(k,\ T)/\partial k}{G_k} \\ &+ \rho \left[-\frac{\partial g_p(k,\ T)/\partial k}{Y_p - g_p + \pi + T} + \alpha \frac{\gamma'(k) g_k(g_p(k,\ T),\ k,\ T) + \partial g_p(k,\ T)/\partial k}{G_p} \right] \\ &+ [v_k'(a(k)) + \rho v_p'(a(k))] a'(k) \le 0 \quad \text{(equality holds if } k > 0). \end{aligned} \tag{27}$$

The child's location in the equilibrium $k^* = k(T)$ is obtained from (27) as a function of the level of public pensions.

The child may choose to live with the parents in the same home: $k^* = 0$, where k^* is the child's location in the equilibrium. We examine the condition by evaluating dU_k/dk for $k = 0$, namely the change in the child's utility when she moves from her parents' home. From $\pi = 0,\ g_p = 0$ (Proposition 1) and $\partial g_p/\partial k = 0$, we have

$$\left.\frac{dU_k}{dk}\right|_{k=0} = \frac{Y_k'(0)}{Y_k(0) - g_k - T} + \rho\alpha\gamma'(0) + [v_k'(a(0)) + \rho v_p'(a(0))] a'(0). \tag{28}$$

Substituting (21) into (28) yields[5]

$$\left.\frac{dU_k}{dk}\right|_{k=0} = \frac{(1-\theta) Y_k'(0)}{C_k} + \alpha \frac{\theta Y_k'(0)}{G_k} + \rho\alpha \frac{\theta Y_k'(0) + \gamma'(0) g_k}{G_p} + [v_k'(a(0)) + \rho v_p'(a(0))] a'(0). \tag{29}$$

Given that, when $\pi = 0$ and $g_p = 0$, θ and $1-\theta$ represent the child's marginal (average) propensity to expend on family public goods, and that to expend on private goods, respectively, (29) means the following.[6] The first and second terms in (29),

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[5] The derivation of (29) is shown in Appendix.

[6] We have $g_k = \theta(Y_k(0) - T) - (1-\theta) g_p$ from (9) with $k = 0\ (\gamma = 1)$ and $\pi = 0$. Therefore, when

$Y_k'(0)(1-\theta)/C_k + \alpha\theta Y_k'(0)/G_k$ (>0), represent the effect through the increases in the child's consumption of private and family public goods in response to the increase in her income when she moves closer to the central business district. The third term, $\rho\alpha[\theta Y_k'(0)+\gamma'(0)g_k]/G_p$, represents the effect through the change in the parents' consumption of family public goods. While the child raises the expenditure on family public goods g_k as her income rises, the spill-over effect of g_k on G_p is weaker when the child lives away from the parents.[7] Since the child's location has these two opposite effects on G_p, the sign of the third term is indeterminate. The fourth term, $[v_k{}'(a(0))+\rho v_p{}'(a(0))]a'(0)$ (<0), represents the effect through the decrease in attention when the child moves away from the parents. Therefore, the sign of (29) is indeterminate, and if $(29)\leq 0$, then $k^*=0$ (alternatively, if $(29)>0$, then $k^*>0$).

We next examine the effect of public pensions on the child's location choice. As a starting point of the analysis, we suppose an equilibrium where, given an arbitrary level of public pensions, the parents and child live in the same home. Namely, $(29)\leq 0$ holds. This can be divided into two cases: $k=0$ is the corner solution and it is the interior solution. We first examine the effect of an increase in T on k for the case that $k=0$ is the corner solution. Differentiating (29) with respect to T and noting $-1<\partial g_k/\partial T=-\theta<0$ obtained from (21) yields

$$(30)\left(\frac{\partial}{\partial T}\right)\left(\frac{dU_k}{dk}\right)\Bigg|_{k=0}=\frac{(1-\theta)Y_k'(0)}{(C_k)^2}\left(\frac{\partial g_k}{\partial T}+1\right)-\left[\frac{\alpha\theta Y_k'(0)}{G_k^2}+\rho\alpha\left(\frac{\theta Y_k'(0)}{G_p^2}\right)\right]\left(\frac{\partial g_k}{\partial T}\right)>0.$$

An increase in T raises the marginal utility of C_k, G_k, G_p, and thus $(dU_k/dk)_{k=0}$. This is because an increase in T depresses the child's expenditure on private goods and family public goods, leading to the decrease in C_k, G_k and G_p.[8]

Since $(dU_k/dk)_{k=0}<0$ holds for corner solution, (30) implies that , when T

$g_p=0$, θ represents the child's marginal (average) propensity to expend on family public goods.

[7] Since $g_p=0$, we have no spill-over effect of g_p on G_k.

[8] While the parental consumption C_p increases as the level of public pensions rises, C_p does not depend on k because $g_p=\pi=0$.

increases and reaches to a certain level, $k(T)=0$ should be obtained as the interior solution (namely, $(dU_k/dk)_{k=0}=0$ should hold for a certain level of T). We denote such a level of T as $\hat{T}$. Given $T=\hat{T}$, differentiating (29) with respect to k and T yields

$$(31)\ \frac{dk(T)}{dT}\bigg|_{T=\hat{T}}=\frac{-1}{\hat{D}}\left\{\frac{(1-\theta)Y_k'(0)}{(C_k)^2}\left(\frac{\partial g_k}{\partial T}+1\right)+\left[\frac{-\alpha\theta Y_k'(0)}{G_k^2}+\rho\alpha\left(\frac{-\theta Y_k'(0)}{G_p^2}\right)\right]\left(\frac{\partial g_k}{\partial T}\right)\right\}>0,$$

where $\hat{D}=d^2U_k/dk^2<0$. From (30) and (31), we obtain the following proposition:[9]

Proposition 2. Suppose that, given an arbitrary level of public pensions, the parents and child live in the same home. If the level of public pensions rises and surpasses $\hat{T}$, the child moves from the parents.

4. Effect of public pensions on welfare

In the previous section, we showed that the parents and child live in the same home when the level of public pensions is less than $\hat{T}$, but the child moves from the parents when it surpasses $\hat{T}$. In this section, taking the change in the child's choice of location into consideration, we examine the effect of public pensions on the child's and the parents' welfare.

4.1 Effect of public pensions on child's welfare

The indirect utility function for the child is give by

$$\begin{aligned}W_k(T)=&\log(Y_k(k(T))-g_k-\pi-T)+\alpha\log G_k+v_k(a(k(T)))\\&+\rho[\log(Y_p-g_p+\pi+T)+\alpha\log G_p+v_p(a(k(T)))],\end{aligned}$$

where, $g_k=\tilde{g}_k(T)$, $\pi=\tilde{\pi}(T)$, $g_p=\tilde{g}_p(T)$, $G_k=\tilde{g}_k(T)+\gamma(k(T))\tilde{g}_p(T)$, and $G_p=\tilde{g}_p(T)+\gamma(k(T))\tilde{g}_k(T)$.[10]

---

[9] If either π or g_p is positive in the initial equilibrium, public pensions are neutral and irrelevant for the child's choice of location. This is apparent from Warr's (1983) neutrality theorem and Ricardian equivalence theorem (Barro, 1974).

[10] $\tilde{\pi}(T)$, $\tilde{g}_k(T)$ and $\tilde{g}_p(T)$ are obtained by substituting $k=k(T)$ into the reaction functions (10),

When $k=0$, from $g_p=0$ and $\pi=0$ (Proposition 1), we have

$$
\begin{aligned}
W_k(T) &= \log(Y_k(k(T)) - g_k - T) + \alpha \log g_k + v_k(a(k(T))) \\
&\quad + \rho[\log(Y_p + T) + \alpha \log \gamma(k(T)) g_k + v_p(a(k(T)))],
\end{aligned} \tag{32}
$$

where $g_k = \theta[Y_k(k(T)) - T]$.

Differentiating (32) with respect to T, using the envelope theorem, and evaluating for $T=\hat{T}$ yields[11]

$$
\left.\frac{dW_k(T)}{dT}\right|_{T=\hat{T}} = -\frac{1}{Y_k(0) - g_k - \hat{T}} + \frac{\rho}{Y_p + \hat{T}}. \tag{33}
$$

From (8) with strict inequality, we have (33)<0, which proves the following proposition.

Proposition 3. A marginal increase in public pensions from $\hat{T}$ reduces the welfare of the child.

4.2 Effect of public pensions on parents' welfare

The indirect utility function for the parents is given by

$$
W_p(T) = \log(Y_p - g_p + \pi + T) + \alpha \log G_p + v_p(a(k(T))).
$$

When $k=0$, form $g_p=0$ and $\pi=0$ (Proposition 1), we have

$$
W_p(T) = \log(Y_p + T) + \alpha \log \gamma(k(T)) g_k + v_p(a(k(T))). \tag{34}
$$

Differentiating (34) with respect to T and evaluating for $T=\hat{T}$ yields[12]

$$
\left.\frac{dW_p(T)}{dT}\right|_{T=\hat{T}} = \frac{1}{Y_p + \hat{T}} - \frac{\alpha}{Y_k(0) - \hat{T}} + \left\{ \frac{\alpha}{G_p}[\theta Y_k'(0) + \gamma'(0) g_k] + v_p'(a(0)) a'(0) \right\} \left.\frac{dk}{dT}\right|_{T=\hat{T}}. \tag{35}
$$

The first and second terms in (35), $[1/(Y_p+\hat{T})] - [\alpha/(Y_k(0)-\hat{T})]$, represent the direct effect through the change in the parents' and child's disposal income by the public pensions. While the parents' consumption of private goods increases in response to the increase in the parents' disposal income, the parents' consumption of family public goods decreases in response to the decrease in the child's disposal income. The sign of

(11) and (17).

[11] See Appendix for the derivation of (33).

[12] See Appendix for the derivation of (35).

the sum of the first and second terms is always positive from the sufficient condition in Proposition 1. However, as the difference between $Y_k(0)-\hat{T}$ and $Y_p+\hat{T}$ is smaller, this direct effect diminishes.

On the other hand, the third term, $(\alpha / G_p)[\theta Y_k'(0)+\gamma'(0)g_k]$, represents the effect through the parents' consumption of family public goods and the fourth term $v_p'(a(0))a'(0)$ represents the effect through the attention. Both of them are the indirect effects through the change in the child's location $dk/dT|_{T=\hat{T}}$. When the child moves away from the parents, the parents' consumption of family public goods increases by $\theta Y_k'(0)$ through the increase in g_k in response to the increase in her income; but at the same time, it decreases by $\gamma'(0)g_k$ through the decrease in the spill-over effect. The attention decreases with the child's moving away from the parents. Using the necessary and sufficient condition for the child living with her parents in the same home ($(29)\leq 0$), the sum of the third and fourth terms becomes negative,[13] implying that the indirect effect on the parents' welfare through the child's location choice is negative. The larger $|\gamma'(0)|$ and $|a'(0)|$ are, the stronger the indirect effect is. Moreover, from (30), as $Y_k'(0)$ rises, $dk/dT|_{T=\hat{T}}$ rises (i.e., the child moves further away from the parents), enhancing the indirect effect. If the indirect effect dominates the direct effect, we have (35)<0. Thus, we obtain the following proposition:

Proposition 4. If $Y_p+\hat{T}$ are close enough to $Y_k(0)-\hat{T}$, or, if $Y_k{'}(0)$, $|\gamma'(0)|$ and $|a'(0)|$ are large enough, then a marginal increase in public pensions from $\hat{T}$ reduces the welfare of the parents.

Propositions 3 and 4 derive the following proposition:

Proposition 5. If one of the sufficient conditions in Proposition 4 is satisfied, then public pensions make both the parents and child worse off through the child's location choice.

---

[13] See Appendix.

5. Optimal level of public pensions

In this section, we derive the optimal level of public pensions in our model. Proposition 5 implies that, if one of the sufficient conditions in Proposition 4 holds, the optimal level of the public pensions is lower than $\hat{T}$. In addition, we can infer from Propositions 3 and 4 that Pareto efficiency is attained for any T lower than $\hat{T}$, because a marginal increase in public pensions does not affect the child's location choice and enhances the parents' welfare (while reducing the child's). Therefore, the optimal level of public pensions should depend on the weight attached to the parents' and the child's utility in the social welfare function.

In this section, it is assumed that the sufficient condition in Proposition 1 holds for $T = 0$. Namely, we assume

(36) $$\rho(1-\theta) < Y_p / Y_k(0) < 1/\alpha .$$

$\rho(1-\theta) < Y_p / Y_k(0)$ implies that the child makes no private income transfers to her parents even in the absence of public pensions, when the child lives with her parents in the same home. [14]

First, we show Pareto efficiency for $T < \hat{T}$. Differentiating (32) with respect to T and noting that k takes a corner solution ($dk/dT = 0$), we have the change in the child's welfare caused by an increase in public pensions as

(37) $$\left.\frac{dW_k(T)}{dT}\right|_{T<\hat{T}} = -\frac{1}{Y_k(0) - g_k - T} + \frac{\rho}{Y_p + T}.$$

From (8) with strict inequality, (37) is negative. As to the change in the parents' welfare, differentiating (34) with respect to T and noting that k takes the corner solution ($dk/dT = 0$), we have

(38) $$\left.\frac{dW_p(T)}{dT}\right|_{T<\hat{T}} = \frac{1}{Y_p + T} - \frac{\alpha}{Y_k(0) - T}.$$

---

[14] If $\rho(1-\theta) > Y_p / Y_k(0)$, the child transfers income to her parents in the absence of public pensions, even when they live in the same home. However, the result on the optimal public pensions is similar to the case in which (36) hold. See Appendix.

From the sufficient condition in Proposition 1, (38) is positive. While the parents consume more private goods, their consumption of family public goods decreases because the child reduces her contribution to family public goods in response to the increase in T. The latter effect dominates the former when the non-negativity constraint on g_p is binding. Thus, Pareto efficiency is achieved for any $T\ (<\hat{T})$.

We are now in a position to derive the optimal level of public pensions. We define the social welfare function according to Blumkin and Sadka (2003) as follows:

$$W(T;\beta) = N[W_k(T) + \beta W_p(T)], \tag{39}$$

where N is the number of families and $\beta\ (\beta \geq 0)$ is the weight attached to parents' utility by the government. The government determines the optimal level of public pensions so as to maximize the social welfare function.[15] Noting that T is nonnegative and using (19) and (21), the first-order condition for this problem is

$$\frac{dW(T;\beta)}{dT}T = N\left[\frac{-[1+\alpha(1+\rho)]}{Y_k(0)-T} + \frac{\rho+\beta}{Y_p+T} + \frac{-\alpha\beta}{Y_k(0)-T}\right]T = 0. \tag{40}$$

We first consider the case of $\beta = 0$. Noting (19), the sufficient condition in Proposition 1 implies

$$\left.\frac{dW(T;\beta)}{dT}\right|_{\beta=0} = N\left[\frac{-(Y_p+T)[1+\alpha(1+\rho)] + \rho(Y_k(0)-T)}{(Y_k(0)-T)(Y_p+T)}\right] < 0 \tag{41}$$

From (40) and (41), we have $T = 0$ if $\beta = 0$.

We turn to the case of $\beta > 0$. Differentiating (41) with respect to β yields

$$\frac{\partial}{\partial\beta}\frac{dW(T;\beta)}{dT} = N\left[\frac{(Y_k(0)-T) - \alpha(Y_p+T)}{(Y_p+T)(Y_k(0)-T)}\right], \tag{42}$$

which is positive under the sufficient condition in Proposition 1. We also have $d^2W/dT^2 < 0$, so that, raising β incrementally from zero, we attain the level of β which satisfies $(dW(T;\beta)/dT)_{T=0} = 0$. Denoting this level of β as $\hat{\beta}$, the optimal

[15] As in Blumkin and Sadka (2003), we can consider several cases according to the value of β. When $\beta = 1$, the social welfare function assigns equal weights to the parents' and the child's welfare, and double-counts the *private* parents' welfare. When $\beta = 1-\rho$, the *private* parents' welfare is 'laundered out' of the child's welfare, which implies that the social welfare function assigns equal weights to the *private* parents' and the *private* child's welfare. When $\beta = 0$, we have $W(T) = W_k(T)$, that is, the government counts the parents' welfare only through the child's.

level of T is positive for $\beta > \hat{\beta}$. Therefore, (40) yields

(43) $$T^*(\beta) = \frac{(\rho+\beta)Y_k(0) - [1+\alpha(1+\rho+\beta)]Y_p}{(1+\alpha)(1+\rho+\beta)},$$

where $T^*(\beta)$ is the optimal level of T depending on β. Differentiating (43) with respect to β yields

(44) $$\frac{dT^*(\beta)}{d\beta} = \frac{Y_k(0)+Y_p}{(1+\alpha)(1+\rho+\beta)^2} > 0.$$

We thus obtain the following proposition:

Proposition 6. (i) If $0 \le \beta \le \hat{\beta}$, then the optimal level of public pensions is zero,
(ii) If $\beta > \hat{\beta}$, then the optimal level of public pensions is positive and increasing in β,
where $\hat{\beta} = \frac{-\rho Y_k(0) + [1+\alpha(1+\rho)]Y_p}{Y_k(0) - \alpha Y_p} (>0).$

To examine how likely the optimal level of public pensions is to be positive, we now suppose $\beta = 1-\rho$, which is a plausible case in that the parents' welfare is 'laundered out' of the child's welfare and the social welfare function assigns equal weights to the private child's and the private parents' welfare. The result is summarized in the following corollary.

Corollary 1 (Optimal public pensions in the case of 'laundered out': $\beta = 1-\rho$).
If $Y_p / Y_k(0) < 1/(2\alpha+1)$, then the optimal level of public pensions is positive.

Proof:

We have $1-\rho-\hat{\beta} = [Y_k(0) - (2\alpha+1)Y_p]/[Y_k(0) - \alpha Y_p]$. This implies that if $Y_p / Y_k(0) < 1/(2\alpha+1)$, then $\hat{\beta} < 1-\rho$. From Proposition 6, therefore, we have $T^*(\beta) > 0$ when $\beta = 1-\rho$. □

For example, under $\alpha = 1$, the sufficient condition in Corollary 1 is reduced to $3Y_p < Y_k(0)$. In Japan, approximately 70% of the income of the retired generation (65 years old or older) comes from public pension benefits (Ministry of Health, Labor and Welfare, 2010). Therefore, the income of the working generation on average is far more than three times the income of the retired generation other than public pension benefits. This implies that, under reasonable values of parameters Y_p, $Y_k(0)$, α and β, the optimal level of public pensions is likely to be positive, even if an increase in T from $\hat{T}$ makes both the parents and child worse off through the child's moving away from the parent.

6. Conclusion

This paper attempted to examine the relevance of a commonly held view that the welfare state or social security tends to loosen family bonds, in other words, to decrease attention or the care children provide to their parents. For this purpose, we explicitly considered the location choice of the child because the feasible level of attention should be subject to the distance between the child's and the parent's residence. Since parents require attention as they become old, weaker family bonds may result in lower family welfare. Therefore, we also analyzed how public pensions affect the parents' and child's welfare through this channel, as well as through intergenerational income redistribution, which public pensions are intended to bring about. In this analysis, financial support from the child to the parents in two ways, income transfers and provision of family public goods, also played a crucial role.

The main results obtained in this paper are as follows. First, if the parents and child live in the same home under a certain level of public pensions, the child transfers no income to the parents while paying everything for family public goods under a plausible condition. Second, even if the child lives with the parents under a certain level of public pensions, the child chooses to live away from the parents as the level of public pensions rises. Third, the child's moving away from the parents due to the increase in public pensions decreases the child's welfare, and may increase or decrease the parents' welfare, depending on the relative magnitude between the direct and indirect effects of public pensions (the latter is results from the child's location choice). If the latter dominates the former, the increase in public pension is Pareto disimproving. Fourth, in spite of the third result, the optimal level of public pensions is positive under reasonable parameter values.

Appendix

1. Derivation of (12)－(14) and (18).

Differentiating (8) and (9) with respect to π, g_k and g_p yields

(A1)
$$\begin{bmatrix} \frac{-1}{C_k^2}+\frac{-\rho}{C_p^2}, & \frac{-1}{C_k^2} \\ \frac{-1}{C_k^2}, & \frac{-1}{C_k^2}+\frac{-\alpha}{G_k^2}+\frac{-\rho\alpha\gamma^2}{G_p^2} \end{bmatrix} \begin{bmatrix} d\pi \\ dg_k \end{bmatrix} = \begin{bmatrix} \frac{-\rho}{C_p^2} \\ \frac{\gamma\alpha}{G_k^2}+\frac{\rho\alpha\gamma}{G_p^2} \end{bmatrix}.$$

From (A1), we have

$$\frac{\partial\pi^+}{\partial g_p} = \frac{\begin{vmatrix} \frac{-\rho}{C_p^2}, & \frac{-1}{C_k^2} \\ \frac{\gamma\alpha}{G_k^2}+\frac{\rho\alpha\gamma}{G_p^2}, & \frac{-1}{C_k^2}+\frac{-\alpha}{G_k^2}+\frac{-\rho\alpha\gamma^2}{G_p^2} \end{vmatrix}}{D}.$$

Adding the second column to the first column of the determinant yields

$$\frac{\partial\pi^+}{\partial g_p} = \frac{\begin{vmatrix} \frac{-1}{C_k^2}+\frac{-\rho}{C_p^2}, & \frac{-1}{C_k^2} \\ \frac{-1}{C_k^2}+\alpha(1-\gamma)\left(\frac{-1}{G_k^2}+\frac{\rho\gamma}{G_p^2}\right), & \frac{-1}{C_k^2}+\frac{-\alpha}{G_k^2}+\frac{-\rho\alpha\gamma^2}{G_p^2} \end{vmatrix}}{D}$$

$$=1+\frac{\alpha(1-\gamma)}{D}\frac{1}{C_k^2}\left(\frac{-1}{G_k^2}+\frac{\rho\gamma}{G_p^2}\right)>0.$$

Thus, we obtain (12). Similarly, from (A1) we have

$$\frac{\partial g_k^+}{\partial g_p} = \frac{\begin{vmatrix} \frac{-1}{C_k^2}+\frac{-\rho}{C_p^2}, & \frac{-\rho}{C_p^2} \\ \frac{-1}{C_k^2}, & \frac{\alpha\gamma}{G_k^2}+\frac{\rho\alpha\gamma}{G_p^2} \end{vmatrix}}{D}.$$

Multiplying the second column of the determinant by (-1) and adding the first column yields

$$\frac{\partial g_k^+}{\partial g_p} = \frac{-\begin{vmatrix} \frac{-1}{C_k^2} + \frac{-\rho}{C_p^2}, & \frac{-1}{C_k^2} \\ \frac{-1}{C_k^2}, & \frac{-1}{C_k^2} - \frac{\alpha\gamma}{G_k^2} - \frac{\rho\alpha\gamma}{G_p^2} \end{vmatrix}}{D}$$

$$= \frac{-\begin{vmatrix} \frac{-1}{C_k^2} + \frac{-\rho}{C_p^2}, & \frac{-1}{C_k^2} \\ \frac{-1}{C_k^2}, & \frac{-1}{C_k^2} + \frac{-\alpha}{G_k^2} + \frac{-\rho\alpha\gamma^2}{G_p^2} + \alpha(1-\gamma)\left(\frac{1}{G_k^2} + \frac{-\rho\gamma}{G_p^2}\right) \end{vmatrix}}{D}$$

$$= -1 + \frac{\alpha(1-\gamma)}{D}\left(\frac{1}{C_k^2} + \frac{\rho}{C_p^2}\right)\left(\frac{1}{G_k^2} - \frac{\rho\gamma}{G_p^2}\right) < 0.$$

Thus, we obtain (13).

When $\pi = 0$, differentiating (9) with respect to g_k and g_p yields

$$\frac{-1}{C_k^2} dg_k + \frac{-\alpha}{G_k^2}(dg_k + \gamma dg_p) + \frac{-\rho\alpha\gamma}{G_p^2}(dg_p + \gamma dg_k) = 0.$$

Thus, we obtain (14). When $\gamma = 1$, we have $G = G_K = G_p = g_k + g_p$, and (14) is rewritten as follows:

(A2) $$\frac{\partial g_k^0}{\partial g_p} = \frac{-\alpha(1+\rho)/G^2}{(1/C_k^2) + [\alpha(1+\rho)/G^2]}.$$

From (9) with $\gamma = 1$, we have $1/C_k = \alpha(1+\rho)/G$. Substituting this equation into (A2) yields (18).

2. Derivation of (29)

Substituting (21) into (28) yields

(A3) $$\left.\frac{dU_k}{dk}\right|_{k=0} = \frac{\theta Y_k'(0)}{g_k(1-\theta)} + \rho\alpha\gamma'(0) + [v_k'(a(0)) + \rho v_p'(a(0))]a'(0).$$

Furthermore, substituting $1-\theta = 1/[1+\alpha(1+\rho)]$ and $g_k = [\theta/(1-\theta)]C_k$ derived from (21) into the first term on the right-hand side of (A3) and noting $G_k = G_p = g_k$ obtained from $g_p = 0$ and $\gamma = 1$ yield (29).

3. Derivation of (33)

Differentiating (32) with respect to T yields

$$\text{(A4)}\quad \begin{aligned}\frac{dW_k(T)}{dT} &= \frac{1}{Y_k(0)-g_k-T}\left(Y_k'(0)\frac{dk}{dT}-\frac{dg_k}{dT}-1\right)+\frac{\alpha}{g_k}\frac{dg_k}{dT}+v_k'(a(0))a'(0)\frac{dk}{dT} \\ &\quad +\frac{\rho}{Y_p+T}+\frac{\rho\alpha}{\gamma g_k}\left(\gamma'(0)g_k\frac{dk}{dT}+\gamma\frac{dg_k}{dT}\right)+\rho v_p'(a(0))a'(0)\frac{dk}{dT} \\ &= \frac{-1}{Y_k(0)-g_k-T}+\frac{\rho}{Y_p+T}+\left(\frac{-1}{Y_k(0)-g_k-T}+\frac{\alpha}{g_k}+\frac{\rho\alpha}{g_k}\right)\frac{dg_k}{dT} \\ &\quad +\left\{\frac{Y_k'(0)}{Y_k(0)-g_k-T}+\frac{\rho\alpha}{\gamma g_k}\gamma'(0)g_k+[v_k'(a(0))+\rho v_p'(a(0))]a'(0)\right\}\frac{dk}{dT},\end{aligned}$$

where $dg_k/dT=\theta\left[Y_k'(0)(dk/dT)-1\right]$. When $k=0$ $(\gamma=1)$, from (9) the third term on the right-hand side of (A4) is 0 . Furthermore, since (28) is 0 when $k=0$ as the interior solution, the fourth term on the right-hand side of (A4) is also 0. Therefore, we obtain (33).

4. Derivation of (35)

Differentiating (34) with respect to T and evaluating the resulting equation for $T=\hat{T}$ yields

$$\text{(A5)}\quad \left.\frac{dW_p(T)}{dT}\right|_{T=\hat{T}}=\frac{1}{Y_p+\hat{T}}+\frac{\alpha}{G_p}\left.\frac{dG_p}{dT}\right|_{T=\hat{T}}+v_p'(a(0))a'(0)\left.\frac{dk}{dT}\right|_{T=\hat{T}}.$$

Noting $G_p=\gamma(0)g_k=\gamma(0)\theta[Y_k(0)-\hat{T}]$, we have

$$\text{(A6)}\quad \left.\frac{dG_p}{dT}\right|_{T=\hat{T}}=\gamma'(0)\theta[Y_k(0)-\hat{T}]\left.\frac{dk}{dT}\right|_{T=\hat{T}}+\theta[Y_k'(0)\left.\frac{dk}{dT}\right|_{T=\hat{T}}-1].$$

Substituting (A6) into (A5) and noting $\alpha\theta/G_p=\alpha\theta/g_k=\alpha/[Y_k(0)-\hat{T}]$ and $G_p=\theta[Y_k(0)-\hat{T}]$ obtained from (21) yields

$$\begin{aligned}\left.\frac{dW_p(T)}{dT}\right|_{T=\hat{T}} &= \frac{1}{Y_p+\hat{T}}+\frac{\alpha}{G_p}[\gamma'(0)G_p+\theta Y_k'(0)]\left.\frac{dk}{dT}\right|_{T=\hat{T}}-\frac{\alpha\theta}{G_p}+v_p'(a(0))a'(0)\left.\frac{dk}{dT}\right|_{T=\hat{T}} \\ &= \frac{1}{Y_p+\hat{T}}-\frac{\alpha}{Y_k(0)-\hat{T}}+\left\{\frac{\alpha}{G_p}[\theta Y_k'(0)+\gamma'(0)G_p]+v_p'(a(0))a'(0)\right\}\left.\frac{dk}{dT}\right|_{T=\hat{T}}.\end{aligned}$$

5. Proof that the sum of the third and the fourth terms in (35) is negative.

From (A3), the necessary and sufficient condition for the child to live with her parents in the same home is obtained as

$$\text{(A7)} \qquad \frac{\theta Y_k'(0)}{g_k(1-\theta)} + \rho\alpha\gamma'(0) + [v_k'(a(0)) + \rho v_p'(a(0))]a'(0) \le 0,$$

which is equivalent to $(29) \le 0$. Adding $\gamma'(0)g_k / g_k(1-\theta)$ to the both sides of (A7) yields

$$\text{(A8)} \quad \theta Y_k'(0) + \gamma'(0)g_k \le [1-\rho\alpha(1-\theta)]\gamma'(0)g_k - (1-\theta)[v_k'(a(0)) + \rho v_p'(a(0))]a'(0)g_k.$$

From (A8), $v_p'(a(0)) > 0$ and $v_k'(a(0)) < 0$ imply

$$\frac{\alpha}{g_k}\left[\theta Y_k'(0) + \gamma'(0)g_k\right] + v_p'(a(0))a'(0) \le \alpha[1-\rho\alpha(1-\theta)]\gamma'(0)$$
$$- \alpha(1-\theta)v_k'(a(0))a'(0) + (1-\rho\alpha)(1-\theta)v_p'(a(0))a'(0) < 0. \qquad \square$$

6. Optimal public pensions in the case of $Y_p / Y_k(0) < \rho(1-\theta)$ $(< 1/\alpha)$.

In this case, we have $\pi > 0$ and $g_p = 0$ when $T = 0$. This implies that $dg_k = 0$ and $d\pi = -dT$ are derived from (8) and (9). Therefore, as T increases from zero, $\pi(>0)$ decreases by the same amount and equals to zero at $\tilde{T} = [\rho(1-\theta)Y_k(0) - Y_p]/[1+\rho(1-\theta)]$, which satisfies $\rho(1-\theta) = (Y_p + T)/[Y_k(0) - T]$. Since $dg_k = 0$ and $d\pi = -dT$, the parents' and child's utility are constant, and so is the social welfare for any $T \in [0, \tilde{T}]$. On the other hand, we have $\pi = 0$ when $T > \tilde{T}$. Therefore, the analysis in Section 5 can be applied. We have $dW(T;\beta)/dT < 0$ for any $T > \tilde{T}$ if $0 \le \beta \le \hat{\beta}$, while there exists $T(>\tilde{T})$ such that $dW(T;\beta)/dT = 0$ if $\beta > \hat{\beta}$. This leads to the following result:

(i) If $0 \le \beta \le \hat{\beta}$, then $T^*(\beta) \in \mathrm{T}, \mathrm{T} = \{T \mid 0 \le T \le \tilde{T}\}$ (indeterminacy arises for $T^*(\beta)$)

(ii) If $\beta > \hat{\beta}$, then $T^*(\beta) > 0$.

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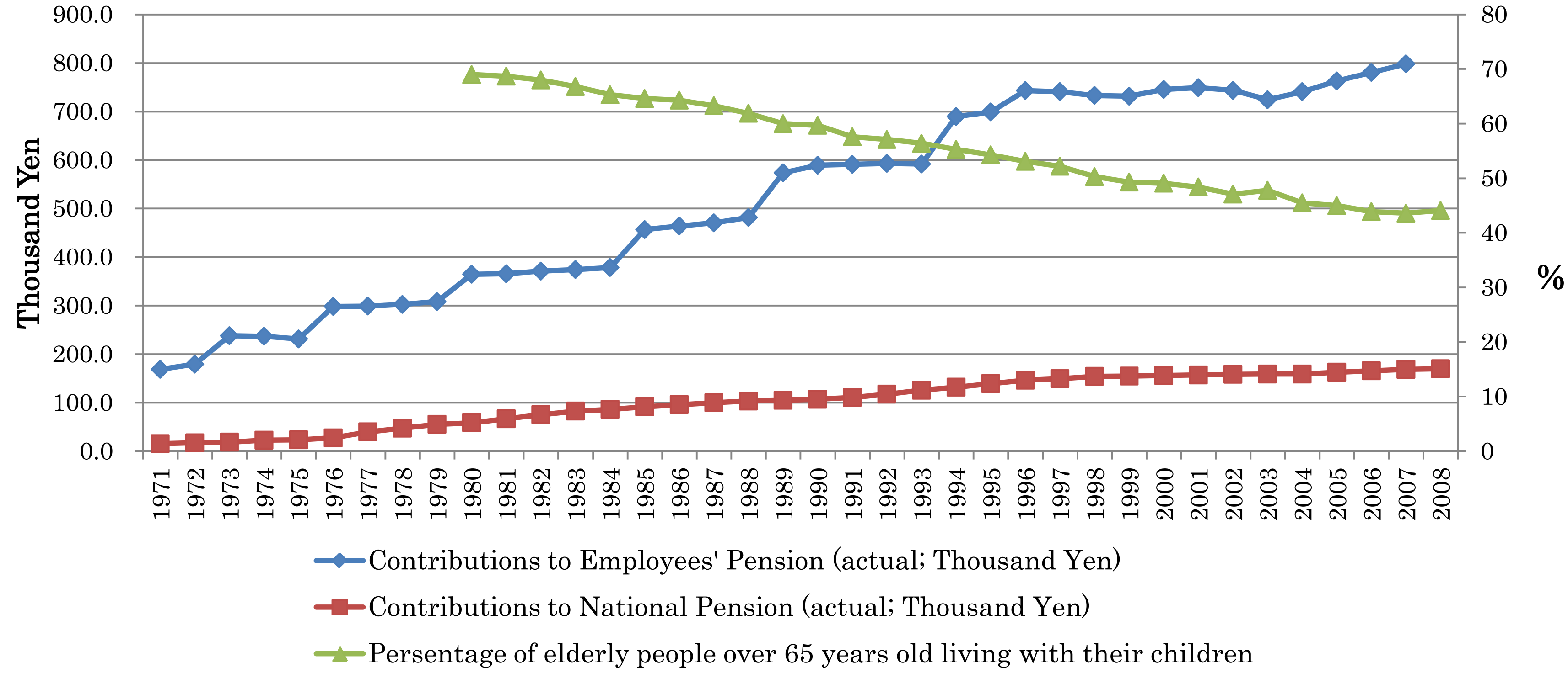
Figure 1. Percentage of elderly people living with their children and average amounts of contributions to public pensions in Japan.



Source: Ministry of Health, Labour and Welfare (Japan), *Comprehensive Survey of Living Conditions*; Social Insurance Agency (Japan), *Annual Report on Social Insurance Services*.

Notes: Contributions to Employees' Pension are calculated by (average standard monthly remuneration)×(contribution rate)×12 before 2002, and {(average standard monthly remuneration)+(average standard bonus amount)/6}×contribution rate×12 after 2003. All amounts are deflated by CPI.